Algebra 1

Unit 3: Quadratic Functions

Romeo High School

Contributors:

Jennifer Boggio
Jennifer Burnham
Jim Cali
Danielle Hart

Robert Leitzel
Kelly McNamara
Mary Tarnowski
Josh Tebeau
After successful completion of this unit, you will be able to:

- Find the factors (linear components) of a quadratic given in standard form using the graph, table or algebraically.

- Determine if a function given in tabular form is quadratic by looking at the change in change.

- Determine the domain and range in the context of a given quadratic situation.

- Express quadratic functions in vertex form, factored form and standard form.

- Apply transformation to quadratic functions and represent symbolically.

- Solve quadratic equations and inequalities graphically, with a table or algebraically (including the quadratic formula).

  - Recognize and define the imaginary number \( i \).

- Determine if a given situation can be modeled by a linear function, a quadratic function or neither. If it is a linear or quadratic, write a function to model it.

  - Simplify radicals and solve equations involving radicals.
<table>
<thead>
<tr>
<th>HSCE Code</th>
<th>Expectation</th>
</tr>
</thead>
<tbody>
<tr>
<td>A1.1.1</td>
<td>Give a verbal description of an expression that is presented in symbolic form, write an algebraic expression from a verbal description, and evaluate expressions given values of the variables.</td>
</tr>
<tr>
<td>A1.1.2</td>
<td>Know the definitions and properties of exponents and roots and apply them in algebraic expressions.</td>
</tr>
<tr>
<td>A1.1.3</td>
<td>Factor algebraic expressions using, for example, greatest common factor, grouping, and the special product identities (e.g., differences of squares and cubes).</td>
</tr>
<tr>
<td>A1.2.1</td>
<td>Write and solve equations and inequalities with one or two variables to represent mathematical or applied situations.</td>
</tr>
<tr>
<td>A1.2.2</td>
<td>Associate a given equation with a function whose zeros are the solutions of the equation.</td>
</tr>
<tr>
<td>A1.2.3</td>
<td>Solve linear and quadratic equations and inequalities, including systems of up to three linear equations with three unknowns. Justify steps in the solutions, and apply the quadratic formula appropriately.</td>
</tr>
<tr>
<td>A1.2.8</td>
<td>Solve an equation involving several variables (with numerical or letter coefficients) for a designated variable. Justify steps in the solution.</td>
</tr>
<tr>
<td>A2.1.1</td>
<td>Recognize whether a relationship (given in contextual, symbolic, tabular, or graphical form) is a function and identify its domain and range.</td>
</tr>
<tr>
<td>A2.1.2</td>
<td>Read, interpret, and use function notation and evaluate a function at a value in its domain.</td>
</tr>
<tr>
<td>A2.1.3</td>
<td>Represent functions in symbols, graphs, tables, diagrams, or words and translate among representations.</td>
</tr>
<tr>
<td>A2.1.4</td>
<td>Recognize that functions may be defined by different expressions over different intervals of their domains. Such functions are piecewise-defined (e.g., absolute value and greatest integer functions).</td>
</tr>
<tr>
<td>A2.1.6</td>
<td>Identify the zeros of a function and the intervals where the values of a function are positive or negative. Describe the behavior of a function as ( x ) approaches positive or negative infinity, given the symbolic and graphical representations.</td>
</tr>
<tr>
<td>A2.1.7</td>
<td>Identify and interpret the key features of a function from its graph or its formula(e), (e.g., slope, intercept(s), asymptote(s), maximum and minimum value(s), symmetry, and average rate of change over an interval).</td>
</tr>
<tr>
<td>A2.2.1</td>
<td>Combine functions by addition, subtraction, multiplication, and division.</td>
</tr>
<tr>
<td>A2.2.2</td>
<td>Apply given transformations (e.g., vertical or horizontal shifts, stretching or shrinking, or reflections about the ( x )- and ( y )-axes) to basic functions and represent symbolically.</td>
</tr>
<tr>
<td>A2.2.3</td>
<td>Recognize whether a function (given in tabular or graphical form) has an inverse and recognize simple inverse pairs.</td>
</tr>
<tr>
<td>A2.3.2</td>
<td>Describe the tabular pattern associated with functions having constant rate of change (linear) or variable rates of change.</td>
</tr>
<tr>
<td>A2.4.1</td>
<td>Write the symbolic forms of linear functions (standard [i.e., ( Ax + By = C ), where ( B \neq 0 )], point-slope, and slope-intercept) given appropriate information and convert between forms.</td>
</tr>
<tr>
<td>A2.4.2</td>
<td>Graph lines (including those of the form ( x = h ) and ( y = k )) given appropriate information.</td>
</tr>
<tr>
<td>A2.4.3</td>
<td>Relate the coefficients in a linear function to the slope and ( x )- and ( y )-intercepts of its graph.</td>
</tr>
<tr>
<td>A2.6.1</td>
<td>Write the symbolic form and sketch the graph of a quadratic function given appropriate information (e.g., vertex, intercepts, etc.).</td>
</tr>
<tr>
<td>A2.6.2</td>
<td>Identify the elements of a parabola (vertex, axis of symmetry, and direction of opening) given its symbolic form or its graph and relate these elements to the coefficient(s) of the symbolic form of the function.</td>
</tr>
<tr>
<td>A2.6.3</td>
<td>Convert quadratic functions from standard to vertex form by completing the square.</td>
</tr>
<tr>
<td>A2.6.4</td>
<td>Relate the number of real solutions of a quadratic equation to the graph of the associated quadratic function.</td>
</tr>
<tr>
<td>A2.6.5</td>
<td>Express quadratic functions in vertex form to identify their maxima or minima and in factored form to identify their zeros.</td>
</tr>
<tr>
<td>A3.1.2</td>
<td>Adapt the general symbolic form of a function to one that fits the specifications of a given situation by using the information to replace arbitrary constants with numbers.</td>
</tr>
</tbody>
</table>
| A3.3.1    | Write the symbolic form and sketch the graph of a quadratic function given appropriate
| A3.3.2 | Identify the elements of a parabola (vertex, axis of symmetry and direction of opening) given its symbolic form or its graph and relate these elements to the coefficient(s) of the symbolic form of the function. |
| A3.3.3 | Convert quadratic functions from standard to vertex form by completing the square. |
| A3.3.4 | Relate the number of real solutions of a quadratic equation to the graph of the associated quadratic function. |
| A3.3.5 | Express quadratic functions in vertex form to identify their maxima or minima and in factored form to identify their zeros. |
| L1.1.1 | Know the different properties that hold in different number systems and recognize that the applicable properties change in the transition from the positive integers to all integers, to the rational numbers, and to the real numbers. |
| L2.1.4 | Know that the complex number $i$ is one of two solutions to $x^2 = -1$. |
Double Distributing
Notes
Quadratic Unit
Algebra 1

Simplify by distributing:
1. 5(x – 3) 3. -8(4 – x)

2. -6(2x + 4) 4. (9 + 3x)7

Instead of distributing with one term, we are going to distribute with two terms. This is called **Double Distributing**. We are going to use the **Area Model** to distribute multiple terms.

Example: (x + 5)(x – 2)

<table>
<thead>
<tr>
<th></th>
<th>x</th>
<th>5</th>
</tr>
</thead>
<tbody>
<tr>
<td>x</td>
<td>x²</td>
<td>5x</td>
</tr>
<tr>
<td>-2</td>
<td>-2x</td>
<td>-10</td>
</tr>
</tbody>
</table>

\[ \rightarrow = x^2 + 5x - 2x - 10 \]
\[ = x^2 + 3x - 10 \]

Simplify each of the following:
5. (5x + 2)(x – 6) 7. (x + y)(x + 4)
6. (x – 3)(x – 11) 8. (x – 7)^2
Double Distributing
HW
Quadratic Unit
Algebra 1

Simplify each of the following.
1. \((x - 15)^2\)

6. \((2x + 3)(x + 1)\)

2. \((x + 7)(x - 2)\)

7. \((3x - 10)(3x + 10)\)

3. \((x + 3)(x - 3)\)

8. \((2x - 5)(2x - 5)\)

4. \((3x - 2z)(3y - 7w)\)

9. \((x - 2)(x - 12)\)

5. \((x + 8)^2\)

10. \((x + 8)(x - 9)\)
To factor a number, you need to break up the original number into the smallest prime numbers that can be used to multiply for that number. A **Factor Tree** breaks up a number by smaller multiples using branches from each larger number so that it looks like a tree.

Prime Numbers are numbers that are only divisible by 1 or itself.
(2,3,5,7,11,13,17,19,23,29,37, etc.)

Example: 12  
**Work:**  
\[ 2 \cdot 2 \cdot 3 = 12 \]

**Factorization:**

Draw a factor tree, and then write the factorization.

1. 24  
2. 225

3. \(108x^3y^2\)  
4. \(-70x^2\)

5. \(36xy^2\)  
6. \(63x^2y^2z^3\)
The Greatest Common Factor or GCF is the largest number or variable amount that is shared by ALL sets of numbers. (The largest number that both given values can be divided by.) You can use a factor tree to find all of the multiples each of the two numbers share.

Example: 12, 16

Work:

GCF:

\[
\begin{array}{c}
12 \\
 \quad 2 \\
 \quad 6 \\
 \quad \quad 2 \\
 \quad 3 \\
\end{array} \quad \begin{array}{c}
16 \\
 \quad 4 \\
 \quad \quad 2 \\
 \quad 2 \\
 \quad 2 \\
\end{array} \\
2 \cdot 2 = 4
\]

Find the GCF.
1. 42, 60

2. -18x^2, -54x

3. 24x^2y^2, 66x^4

4. 14a^2b^2, 18ab, 2a^3b^3
Draw a factor tree, and then write the factorization:

1. 90
2. 63
3. $4p^2$
4. $-39b^3c^2$
5. $100x^3yz^2$
6. $-12a^2b^3$
Find the GCF:

7. 27, 72

8. 32, 48

9. 20gh, 36g^2h^2

10. 15r^2s, 35s^2, 70rs
Factoring "Which One Do I Use?" Flow Chart

Notes
Quadratic Unit
Algebra 1

Factoring
*Always check for GCF first*

2 terms

- Is it in the form of $a^2 - b^2$?
  - Yes
    - Use difference of squares of
    factor into
    form $(a + b)(a - b)$
    - Examples
      - $x^2 - 4 = (x+2)(x-2)$
      - $9x^2 - 25 = (3x+5)(3x-5)$
  - No
    - Use GCF to factor; be sure to factor out variables where needed
    - Examples
      - $2x^2 + 18 = 2(x^2 + 9)$
      - $14x^2 - 49x = 7x(2x - 7)$

3 terms

- $a \neq 1$
  - Use the 6 Step Factoring Trinomial Method
    - $ax^2 + bx + c$
  - $a = 1$
  - Multiply the $ax^2$ term and the $c$ term
  - List all of the factors for your answer.
  - Add up all the sets of factors you just found.
  - Choose the set of factors that adds up to the $bx$ term. Rewrite the $bx$ terms as two separate terms
  - Factor by grouping (see 4 terms option)
  - Check your work by double distributing (DD)

4 terms

- Use the grouping method to factor
  - Break into groups, factor and combine. Use DD to check.
  - Example
    - $6x^2 + 3xy + 10x + 5y$
    - $6x^2 + 3xy$
    - $10x + 5y$
    - $3(2x + y)$
    - $5(2x + y)$
    - $(2x + y)(3x + 5)$
    - Double Distribute:
      - $6x^2 + 10x + 3xy + 5y$
    - Find 2 factors of the $c$ term that add up to the $b$ term.
      - Example:
        - $x^2 + 5x + 6 = (x + 2)(x + 3)$
Factoring by Reverse Distributing

Notes

Quadratic Unit

Algebra 1

Simplify by distributing:

1. $6x(x + 2)$  
2. $-7(x - 1)$  
3. $8(x^2 - 5x + 3)$  
4. $x(x^2 - 9)$

Factoring is a process used to break down an expanded expression into non-common terms using parenthesis.

*The Algebra Model:*
- Find the GCF of the terms
- Write the GCF outside of the parenthesis
- Write what is left over inside the parenthesis

Example: $12x^4 - 8x^2 = 4x^2(3x^2 - 2)$

*The Area Model:*
- Write the expression in its factorized form
- Write the GCF on the left
- Simplify the terms
- Rewrite using parenthesis

$$4x^2(3x^2 - 2)$$
1. $36x + 42$

2. $-70x^2 - 20x$

3. $30x + 5$

4. $125x^3 + 225x^4$

5. $16x^2 - 40x$

6. $-12x^2 + 12x$

7. $3x^3 + 12x^2 - 15x$
Factoring by Reverse Distributing

HW
Quadratic Unit
Algebra 1

1. \(8x + 20\)              2. \(-6x^2 - 66x - 168\)

3. \(-3x^2 + 18x\)          4. \(28x + 7\)

5. \(-5x - 25\)             6. \(-2x^3 + 4x^3 + 96x^2\)

7. \(16x^2 - 20x\)          8. \(15x^3 - 375x\)
Factoring by Grouping

Notes

Quadratic Unit

Algebra 1

Factoring by Grouping is a factoring method used when there are four terms. The goal is to break down the expression into two sets of parenthesis.

**Algebra Model**

To factor an expression by grouping using the *algebra model* follow these steps with the example:

\[2x^2 + 8x - 3xy - 12y\]

Step 1: Group the first two terms and the last two terms in parenthesis:

\[(2x^2 + 8x) + (-3xy - 12y)\]

Step 2: Factor each grouped pair by finding the GCF (factoring by reverse distributing):

<table>
<thead>
<tr>
<th>Group 1</th>
<th>Group 2</th>
</tr>
</thead>
<tbody>
<tr>
<td>(2x^2 + 8x)</td>
<td>(-3xy - 12y)</td>
</tr>
<tr>
<td>(2x(x + 4))</td>
<td>(-3y(x + 4))</td>
</tr>
</tbody>
</table>

All factored pieces together:

\[2x(x + 4) + -3y(x + 4)\]

Step 3: The matching parenthesis become one set of parenthesis and the coefficients in front combine to form another set of parenthesis:

\[(x + 4)(2x - 3y)\]

To check your answer, double distribute.

\[(x + 4)(2x - 3y) = 2x^2 + 8x - 3xy - 12y\]

**Area Model**

To factor an expression by grouping using the *area model* follow these steps with the example:

\[2x^2 + 8x - 3xy - 12y\]

Write the four terms in the four boxes on the inside of the boxes:

<p>| | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
</tr>
<tr>
<td>(2x^2)</td>
<td>(8x)</td>
</tr>
<tr>
<td>(-3xy)</td>
<td>(-12y)</td>
</tr>
</tbody>
</table>
Find the GCF for each row and column:

<table>
<thead>
<tr>
<th></th>
<th>x</th>
<th>4</th>
</tr>
</thead>
<tbody>
<tr>
<td>2x</td>
<td>2x^2</td>
<td>8x</td>
</tr>
<tr>
<td>-3y</td>
<td>-3xy</td>
<td>-12y</td>
</tr>
</tbody>
</table>

Write the edges of the model as expressions in parenthesis:

\[(2x - 3y)(x + 4)\]

To check your answer, double distribute.

\[(x + 4)(2x - 3y) = 2x^2 + 8x - 3xy - 12y\]

In-Class Examples:
1. \[8x^2 + 40x + 3xy + 15y\]

2. \[3x^2 + 24x - 7xy - 56y\]

3. \[3x^2 - 27x - xy + 9y\]
1. $6x^2 + 18x - 5xy - 15y$

2. $8x^2 - 8x + xy - y$

3. $200x^3 + 70x^2 - 60x$

4. $5x^2 - 5x - 2xy + 2y$
5. \(4x^2 - 12x + 7xy - 21y\)

6. \(14x^2 - 686\)

7. \(6x^2 + 21x + 8xy + 28y\)

8. \(8x^2 + 18x - 20xy - 45y\)
To factor trinomials, we will use quadratics that are in standard form: \( ax^2 + bx + c \). We will apply the grouping and reverse distributing factoring methods to assist in factoring a trinomial.

**Algebra Model:** \( 2x^2 + 11x + 12 \)

1. Multiply the \( ax^2 \) term and the \( c \) term.
   \[ 2x^2 \cdot 12 = 24x^2 \]

2. List all of the factors for the answer you found for step #1.
   \[
   \begin{array}{c|c}
   1x & 24x \\
   2x & 12x \\
   3x & 8x \\
   4x & 6x \\
   \end{array}
   \]

3. Add up all the sets of factors you found in step #2.
   \[
   \begin{array}{c|c}
   1x & 24x = 25x \\
   2x & 12x = 14x \\
   3x & 8x = 11x \\
   4x & 6x = 24x \\
   \end{array}
   \]

4. Choose the set of factors that adds up to the \( bx \) term; rewrite the \( bx \) term as two separate terms.
   \[ 3x \cdot 8x = 24x^2 \]
   And
   \[ 3x + 8x = 11x \]
   Choose 3x and 8x
   \[
   \text{Rewrite } 2x^2 + \Box \text{ as } 2x^2 + 3x + 8x + 12
   \]

5. Factor by grouping and reverse distributing.
   \[
   \begin{align*}
   &2x^2 + 3x + 8x + 12 \\
   &x(2x + 3) + 4(2x + 3) \\
   & (x + 4)(2x + 3)
   \end{align*}
   \]

6. Check your work by double distributing:
   \[
   \begin{align*}
   &(x + 4)(2x + 3) \\
   &= 2x^2 + 3x + 8x + 12 \\
   &= 2x^2 + 11x + 12
   \end{align*}
   \]
**Area Model:** $2x^2 + 11x + 12$

1. Write the $ax^2$ and $c$ terms in the top left and bottom right corners inside the box.

```
  |   |
---|---|
  | 2x^2 |
  | 12   |
```

2. Use the diamond to determine what multiplies to the $(ax^2 \cdot c)$ term and adds to the $bx$ term.

```
  24x^2
  3x   
  8x   
 11x   
```

3. Write these in the remaining inside boxes.

```
  |   |
---|---|
  | 2x^2 | 3x
  | 8x   | 12
```

4. Factor by grouping.

```
  2x  | 3
  x   | 2x^2 | 3x
  4   | 8x   | 12
```

$=(2x + 3)(x + 4)$

5. Check your work by double distributing: $(x + 4)(2x + 3)$

$= 2x^2 + 11x + 12$
In-Class Work:
1. $x^2 + 13x + 36$
3. $x^2 + 8x + 7$

2. $12x^2 + 29x + 15$
4. $10x^2 + 105x + 135$
Factoring Trinomials #1

Name______________________________

Hour_______ Date ________________

Algebra 1

Factor the following expressions.

1. $2x^2 + 7x + 5$
2. $4x^4 - 64x^2$
3. $x^2 + 12x + 35$
4. $3x^3 + 30x^2 + 48x$
5. $9x^2 + 9x + 2$
6. $10x^3 + 120x^2 + 270x$
7. $x^2 + 13x + 40$
8. $-2x^3 - 16x^2 - 24x$
9. $x^2 + 5x + 4$
10. $28x^2 - 175$
11. $16x^2 + 8x + 1$
12. $12.9x^4 - 72x$
To factor trinomials, we will use quadratics that are in standard form: $ax^2 + bx + c$. We will apply the grouping and reverse distributing factoring methods to assist in factoring a trinomial.

### Algebra Model: $x^2 - 8x + 12$

1. Multiply the $ax^2$ term and the $c$ term. 
   
   $x^2 \cdot 12 = 12x^2$

2. List all of the factors for the answer you found for step #1.

<table>
<thead>
<tr>
<th>Factor 1</th>
<th>Factor 2</th>
<th>Product</th>
</tr>
</thead>
<tbody>
<tr>
<td>1x</td>
<td>12x</td>
<td>12</td>
</tr>
<tr>
<td>-1x</td>
<td>-12x</td>
<td>-12</td>
</tr>
<tr>
<td>2x</td>
<td>6x</td>
<td>12</td>
</tr>
<tr>
<td>-2x</td>
<td>-6x</td>
<td>-12</td>
</tr>
<tr>
<td>3x</td>
<td>4x</td>
<td>12</td>
</tr>
<tr>
<td>-3x</td>
<td>-4x</td>
<td>-12</td>
</tr>
</tbody>
</table>

3. Add up all the sets of factors you found in step #2.

<table>
<thead>
<tr>
<th>Factor 1</th>
<th>Factor 2</th>
<th>Product</th>
</tr>
</thead>
<tbody>
<tr>
<td>1x</td>
<td>12x</td>
<td>12</td>
</tr>
<tr>
<td>-1x</td>
<td>-12x</td>
<td>-12</td>
</tr>
<tr>
<td>2x</td>
<td>6x</td>
<td>12</td>
</tr>
<tr>
<td>-2x</td>
<td>-6x</td>
<td>-12</td>
</tr>
<tr>
<td>3x</td>
<td>4x</td>
<td>12</td>
</tr>
<tr>
<td>-3x</td>
<td>-4x</td>
<td>-12</td>
</tr>
</tbody>
</table>

   $1x - 12x = 13x$
   
   $-1x - -12x = 13x$
   
   $2x + 6x = 8x$
   
   $-2x - 6x = -8x$
   
   $3x + 4x = 7x$
   
   $-3x - 4x = -7x$

4. Choose the set of factors that adds up to the $bx$ term; rewrite the $bx$ term as two separate terms.

   $-2x \cdot -6x = 12x^2$
   
   And
   
   $-2x + -6x = -8x$
   
   Choose $-2x$ and $-6x$

   **Rewrite** $x^2 - 8x + 12$ as $x^2 - 2x - 6x + 12$

5. Factor by grouping and reverse distributing.

   $x^2 - 2x - 6x + 12$
   
   $x(x - 2) + -6(x - 2)$
   
   $(x - 2)(x - 6)$

6. Check your work by double distributing:

   $(x - 2)(x - 6)$
   
   $x^2 - 2x - 6x + 12$
   
   $x^2 - 8x + 12$
**Area Model:** \(x^2 - 8x + 12\)

1. Write the \(ax^2\) and \(c\) terms in the top left and bottom right corners inside the box.

\[
\begin{array}{|c|c|}
\hline
x^2 & \text{ } \\
\hline
\text{ } & 12 \\
\hline
\end{array}
\]

2. Use the diamond to determine what multiplies to the \((ax^2 \cdot c)\) term and adds to the \(bx\) term.

\[
\begin{array}{|c|c|}
\hline
12x^2 & -2x \\
\hline
-8x & \text{ } \\
\hline
\end{array}
\]

3. Write these in the remaining inside boxes.

\[
\begin{array}{|c|c|}
\hline
2x^2 & -2x \\
\hline
-6x & 12 \\
\hline
\end{array}
\]

4. Factor by grouping.

\[
\begin{array}{|c|c|}
\hline
x & -2 \\
\hline
x & x^2 - 2x \\
\hline
-6 & -6x + 12 \\
\hline
\end{array}
\]

\[= (x - 2)(x - 6)\]

5. Check your work by double distributing:

\[(x - 2)(x - 6) = x^2 - 8x + 12\]
In-Class Work:

1. $2x^2 - 9x + 7$
2. $x^2 - 23x + 132$
3. $4x^2 - 40x + 96$
4. $9x^2 - 12x + 4$
Factoring Trinomials #2

HW

Quadratic Unit
Algebra 1

Factor the following expressions.

1. \(8x^2 - 18x + 9\)

7. \(x^2 - 9x + 20\)

2. \(6x^3 - 216x\)

8. \(-4x^3 + 2916\)

3. \(x^2 - 16x + 63\)

9. \(x^2 - 11x + 24\)

4. \(15x^2 - 96x + 36\)

10. \(5x^3 - 75x^2 + 280x\)

5. \(6x^2 - 19x + 15\)

11. \(4x^2 - 4x + 1\)

6. \(8x^5 + 1000x^2\)

12. \(12.4x^2 - 50x + 126\)

RHS Mathematics Department
Algebra 1 Unit 3: Quadratic Functions
Linear, Quadratic Or Neither Flow Chart

Notes

Quadratic Unit
Algebra 1

Take 1st Difference

Same number LINEAR

Different number QUADRATIC / NEITHER

Take 2nd Difference

Same Number QUADRATIC

Different number NEITHER

y = mx + b
form

Find slope (m)
m = Δy
Δx

Find y-intercept (b)
Recall x = 0

Write in the form
y = mx + b

Check equation in graphing calculator by using TABLE.

Find a.
a = \(\frac{2^{nd} \text{ Difference}}{2}\)

Find c.
Plug in a point where x = 0 and solve.

Find b.
Plug in a point where x ≠ 0 and solve.

Write in the form
y = ax^2 + bx + c

Check equation in graphing calculator by using TABLE.
Factor the following expressions.

1. $6x^2 + 5x - 6$
2. $x^2 + 9x + 8$

3. $x^2 - 14x + 45$
4. $x^2 - 19x + 60$

5. $2x^2 - 3x - 20$
6. $6x^2 + 37x + 35$
7. $6x^2 - 14x - 12$

8. $9x^2 - 14x + 5$

9. $x^2 + 9x + 14$

10. $x^2 + 13x - 30$

11. $x^2 - 4x - 32$

12. $2x^2 + 25x + 12$

13. $10x^2 - 17x + 3$

14. $x^2 + x - 42$
Factoring By Difference of Squares

Notes
Quadratic Unit
Algebra 1

Formula: \[ a^2 - b^2 = (a + b)(a - b) \]

Example: \[ x^2 - 81 \] \[ \Rightarrow \sqrt{x^2} = x = a \quad \text{and} \quad \sqrt{81} = 9 = b \]

So plugging \( x \) and 9 into the formula yields \((x + 9)(x - 9)\). Double distribute back to check the result:

\[
(x + 9)(x - 9) \\
= x^2 - 9x + 9x - 81 \\
= x^2 - 81
\]

1. \[ x^2 - 1 \]

2. \[ x^2 - 4 \]

3. \[ 9x^2 - 100 \]

4. \[ x^2 - 16 \]

5. \[ 25x^2 - 36 \]

6. \[ 9x^2 - 1 \]
Factoring By Difference of Squares

HW

Quadratic Unit

Algebra 1

Factor.

1. $x^2 - 25$

6. $16x^2 - 25$

2. $x^2 + 5x - 66$

7. $4x^2 - 81$

3. $x^2 - 64$

8. $9x^2 + 49$

4. $25x^2 - 1$

9. $4x^2 - 36$

5. $16x^2 + 81$

10. $12x^2 - 78x - 270$
11. $4x^2 - 25$

12. $25x^2 - 144$

13. $9x^2 - 121$

14. $16x^2 - 9$

15. $3x^2 + 22x + 7$

16. $25x^2 - 16$

17. $x^3 - 64$

18. $4x^2 - 1$

19. $9x^2 - 4$

20. $x^2 + 10x + 24$

21. $25x^2 - 64$

22. $4x^2 - 144$
Linear, Quadratic Or Neither Flow Chart
Notes
Quadratic Unit
Algebra 1

Take 1st Difference

Same number LINEAR

Different number QUADRATIC / NEITHER

Take 2nd Difference

Same Number QUADRATIC

Different number NEITHER

Find slope (m)

\[ m = \frac{\Delta y}{\Delta x} \]

Find \( y \)-intercept (b)
Recall \( x = 0 \)

Write in the form

\[ y = mx + b \]

Check equation in graphing calculator by using TABLE.

Find \( a \).

\[ a = \frac{2\text{nd Difference}}{2} \]

Find c.
Plug in a point where \( x = 0 \) and solve.

Find b.
Plug in a point where \( x \neq 0 \) and solve.

Write in the form

\[ y = ax^2 + bx + c \]

Check equation in graphing calculator by using TABLE.
A linear function has a constant rate of change which we call \textbf{slope}. In the Linear Unit we found the slope from a table by using \( \frac{\text{change in } y}{\text{change in } x} \) or \( \frac{\Delta y}{\Delta x} \). Find the equation \( y = mx + b \). \textbf{Ex:} \( y = \frac{1}{2} x - 7 \).

**Example 1**

\begin{tabular}{ccc}
\hline
\textbf{x} & \textbf{y} & \textbf{Δy} \\
\hline
0 & -6 & \\
1 & -2 & \\
2 & 2 & \\
3 & 6 & \\
4 & 10 & \\
5 & 14 & \\
\hline
\end{tabular}

\textbf{LINEAR}

Step 1: Find the change in the x’s and y’s:

\textbf{Step 2:} Find the \( y \)–intercept: (when \( x = 0 \))

\textbf{Step 3:} Find the slope using \( \frac{\Delta y}{\Delta x} \):

\textbf{Step 4:} Plug in slope and \( y \)–intercept into the equation \( y = mx + b \):

\textbf{Step 5:} Plug the equation into the calculator and check to make sure all the coordinate points are on the graph or compare the \textbf{TABLE}.

**Example 2**

\begin{tabular}{ccc}
\hline
\textbf{Δx} & \textbf{x} & \textbf{y} & \textbf{Δy} \\
\hline
-2 & 13 & \\
-1 & 10 & \\
0 & 7 & \\
1 & 4 & \\
2 & 1 & \\
3 & -2 & \\
\hline
\end{tabular}
Example 3

<table>
<thead>
<tr>
<th>x</th>
<th>-3</th>
<th>-2</th>
<th>-1</th>
<th>0</th>
<th>1</th>
<th>2</th>
</tr>
</thead>
<tbody>
<tr>
<td>y</td>
<td>-16</td>
<td>-12</td>
<td>-8</td>
<td>-4</td>
<td>0</td>
<td>4</td>
</tr>
</tbody>
</table>

A given set of coordinate points forms a quadratic function when there is a constant $2^{nd}$ difference.
The standard form equation for Quadratics is $y = ax^2 + bx + c$. **Ex:** $y = 3x^2 - 6x + 24$.

Example 4

<table>
<thead>
<tr>
<th>Δx</th>
<th>x</th>
<th>y</th>
<th>$1^{st}$ difference</th>
<th>$2^{nd}$ difference</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>-9</td>
<td></td>
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<td>1</td>
<td>-3</td>
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<td></td>
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<tr>
<td>2</td>
<td>11</td>
<td></td>
<td></td>
<td></td>
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<tr>
<td>3</td>
<td>33</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>4</td>
<td>63</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>5</td>
<td>101</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

**QUADRATIC**

Step 1: Find the change in the x’s and y’s for the $1^{st}$ & $2^{nd}$ difference:

Step 2: Find the y – intercept “c” (when $x = 0$)

Step 3: Find “a” by taking the $2^{nd}$ difference and dividing by 2.

Step 4: Plug in “a”, “c” and any coordinate point besides the y-intercept into the standard form equation to solve for “b”. After, write the equation in the form $y = ax^2 + bx + c$.

Step 5: Put the equation into the calculator and verify all the coordinate points are on the graph or compare the TABLE.
Example 5

<table>
<thead>
<tr>
<th>Δx</th>
<th>x</th>
<th>y</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>12</td>
<td></td>
</tr>
<tr>
<td>1</td>
<td>2</td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>-14</td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>-36</td>
<td></td>
</tr>
<tr>
<td>4</td>
<td>-64</td>
<td></td>
</tr>
<tr>
<td>5</td>
<td>-98</td>
<td></td>
</tr>
</tbody>
</table>

Example 6

<table>
<thead>
<tr>
<th>x</th>
<th>-2</th>
<th>-1</th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
</tr>
</thead>
<tbody>
<tr>
<td>y</td>
<td>5</td>
<td>6</td>
<td>5</td>
<td>2</td>
<td>-3</td>
<td>-10</td>
</tr>
</tbody>
</table>
For now, after looking for the first difference and the second difference, we will call the function “neither.”

**Example 7**

Step 1: Find the changes in x’s and y’s for the 1\textsuperscript{st} and 2\textsuperscript{nd} difference.

<table>
<thead>
<tr>
<th>Δx</th>
<th>x</th>
<th>y</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>-81</td>
<td></td>
</tr>
<tr>
<td>1</td>
<td>-77</td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>-49</td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>27</td>
<td></td>
</tr>
<tr>
<td>4</td>
<td>175</td>
<td></td>
</tr>
<tr>
<td>5</td>
<td>419</td>
<td></td>
</tr>
</tbody>
</table>

1\textsuperscript{st} difference

2\textsuperscript{nd} difference

Step 2: If neither of the differences have a constant, it is neither Linear or Quadratic.

**NEITHER**

**Example 8**

<table>
<thead>
<tr>
<th>x</th>
<th>-1</th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
</tr>
</thead>
<tbody>
<tr>
<td>y</td>
<td>0</td>
<td>3</td>
<td>6</td>
<td>12</td>
<td>24</td>
<td>48</td>
</tr>
</tbody>
</table>
Determine if each function is Linear, Quadratic or Neither. If it is Linear or Quadratic, give the equation.

1.  

<table>
<thead>
<tr>
<th>x</th>
<th>y</th>
<th>1\textsuperscript{st} difference</th>
<th>2\textsuperscript{nd} difference</th>
</tr>
</thead>
<tbody>
<tr>
<td>-2</td>
<td>0</td>
<td></td>
<td></td>
</tr>
<tr>
<td>-1</td>
<td>2</td>
<td></td>
<td></td>
</tr>
<tr>
<td>0</td>
<td>6</td>
<td></td>
<td></td>
</tr>
<tr>
<td>1</td>
<td>12</td>
<td></td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>20</td>
<td></td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>30</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

2.  

<table>
<thead>
<tr>
<th>x</th>
<th>y</th>
</tr>
</thead>
<tbody>
<tr>
<td>-2</td>
<td>-23</td>
</tr>
<tr>
<td>-1</td>
<td>-13</td>
</tr>
<tr>
<td>0</td>
<td>-3</td>
</tr>
<tr>
<td>1</td>
<td>7</td>
</tr>
<tr>
<td>2</td>
<td>17</td>
</tr>
<tr>
<td>3</td>
<td>27</td>
</tr>
</tbody>
</table>

3.  

<table>
<thead>
<tr>
<th>x</th>
<th>y</th>
</tr>
</thead>
<tbody>
<tr>
<td>-2</td>
<td>0</td>
</tr>
<tr>
<td>-1</td>
<td>1</td>
</tr>
<tr>
<td>0</td>
<td>76</td>
</tr>
<tr>
<td>1</td>
<td>441</td>
</tr>
<tr>
<td>2</td>
<td>1456</td>
</tr>
<tr>
<td>3</td>
<td>3625</td>
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</tbody>
</table>
### 4. First and Second Differences

<table>
<thead>
<tr>
<th>$\Delta x$</th>
<th>$x$</th>
<th>$y$</th>
<th>1st difference</th>
<th>2nd difference</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0</td>
<td>-14</td>
<td></td>
<td></td>
</tr>
<tr>
<td>1</td>
<td>1</td>
<td>-15</td>
<td></td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>2</td>
<td>-52</td>
<td></td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>3</td>
<td>-125</td>
<td></td>
<td></td>
</tr>
<tr>
<td>4</td>
<td>4</td>
<td>-234</td>
<td></td>
<td></td>
</tr>
<tr>
<td>5</td>
<td>5</td>
<td>-379</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

### 5. First and Second Differences

<table>
<thead>
<tr>
<th>$\Delta x$</th>
<th>$x$</th>
<th>$y$</th>
<th>1st difference</th>
<th>2nd difference</th>
</tr>
</thead>
<tbody>
<tr>
<td>-1</td>
<td>-1</td>
<td>15</td>
<td></td>
<td></td>
</tr>
<tr>
<td>0</td>
<td>0</td>
<td>10</td>
<td></td>
<td></td>
</tr>
<tr>
<td>1</td>
<td>1</td>
<td>7</td>
<td></td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>2</td>
<td>6</td>
<td></td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>3</td>
<td>7</td>
<td></td>
<td></td>
</tr>
<tr>
<td>4</td>
<td>4</td>
<td>10</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
6. 

\[ \begin{array}{c|c|c|c} \hline x & y & 1^{\text{st}} \text{ difference} & 2^{\text{nd}} \text{ difference} \\ \hline 0 & -5 & & \\ 1 & -5\frac{1}{2} & & \\ 2 & -6 & & \\ 3 & -6\frac{1}{2} & & \\ 4 & -7 & & \\ 5 & -7\frac{1}{2} & & \\ \hline \end{array} \]

7. 

\[ \begin{array}{c|c|c|c|c|c|c} x & -2 & -1 & 0 & 1 & 2 & 3 \\ \hline y & 34 & -4 & -20 & -14 & 14 & 64 \\ \hline \end{array} \]
8. \[ \Delta x \] 

<table>
<thead>
<tr>
<th>( x )</th>
<th>( y )</th>
<th>1st difference</th>
<th>2nd difference</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>-2</td>
<td></td>
<td></td>
</tr>
<tr>
<td>1</td>
<td>-4</td>
<td></td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>-8</td>
<td></td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>-16</td>
<td></td>
<td></td>
</tr>
<tr>
<td>4</td>
<td>-32</td>
<td></td>
<td></td>
</tr>
<tr>
<td>5</td>
<td>-64</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

9. 

<table>
<thead>
<tr>
<th>( x )</th>
<th>-4</th>
<th>-3</th>
<th>-2</th>
<th>-1</th>
<th>0</th>
<th>1</th>
</tr>
</thead>
<tbody>
<tr>
<td>( y )</td>
<td>166</td>
<td>95</td>
<td>42</td>
<td>7</td>
<td>-10</td>
<td>-9</td>
</tr>
</tbody>
</table>

10. \[ \Delta x \] 

<table>
<thead>
<tr>
<th>( x )</th>
<th>( y )</th>
<th>1st difference</th>
<th>2nd difference</th>
</tr>
</thead>
<tbody>
<tr>
<td>-2</td>
<td>-2</td>
<td></td>
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<tr>
<td>-1</td>
<td>.5</td>
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<tr>
<td>0</td>
<td>3</td>
<td></td>
<td></td>
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<tr>
<td>1</td>
<td>5.5</td>
<td></td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>8</td>
<td></td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>10.5</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
To simplify a radical, find the prime factorization of the number under the radical symbol. Then, pairs of numbers are pulled out front of the radical symbol and the “left-over” prime numbers remain under the radical symbol. Simplify all pieces by multiplying. In today’s lesson, the radical can also be called a square root.

Example: \( \sqrt{450} = \sqrt{2 \cdot 3 \cdot 3 \cdot 5 \cdot 5} = 3 \cdot 5\sqrt{2} = 15\sqrt{2} \)

Example: \( 4\sqrt{90} = \sqrt{2 \cdot 3 \cdot 3 \cdot 5} = 4 \cdot 3\sqrt{2 \cdot 5} = 12\sqrt{10} \)

1. \( \sqrt{45} \)  \hspace{1cm} 2. \( \sqrt{256} \)  \hspace{1cm} 3. \( -\sqrt{75} \)  

4. \( \sqrt{100} \)  \hspace{1cm} 5. \( 5\sqrt{10} \cdot 3\sqrt{10} \)  \hspace{1cm} 6. \( 7\sqrt{30} \cdot 2\sqrt{6} \)
Simplifying Radicals, Day 1
HW
Quadratic Unit
Algebra 1

Simplify the radicals.

1. $\sqrt{144}$
2. $-\sqrt{196}$
3. $\sqrt{10} \cdot 2\sqrt{30}$

4. $5\sqrt{100} \cdot 2\sqrt{121}$
5. $\sqrt{20}$
6. $-\sqrt{18}$

7. $\sqrt{280}$
8. $\sqrt{6} \cdot \sqrt{12}$
9. $7 \cdot \sqrt{12}$
Review
For each of the following problems identify what type of factoring is necessary. After, factor completely. (Remember, we have used factor by: Reverse Distributing, Grouping, Trinomials, and Difference of Squares)

10. $4x^2 - 49$ Type: _________________
11. $x^2 - 3x - 10$ Type: _________________
12. $x^2 + 20x + 96$ Type: _________________
13. $x^2 + 2x - xy - 2y$ Type: _________________
**Example** Larry makes a mistake with simplifying and Curly catches the mistake. Larry should have written $6\sqrt{5}$. So Curly steals Larry's 6 points.

<table>
<thead>
<tr>
<th>Player Name</th>
<th>Original Card</th>
<th>Place Mat</th>
<th>“Go Factor”</th>
<th>Simplified</th>
<th>Points Earned</th>
<th>Points Stolen</th>
<th>Total Points</th>
</tr>
</thead>
<tbody>
<tr>
<td>Larry</td>
<td>$\sqrt{180}$</td>
<td>$\sqrt{2 \cdot 2 \cdot 3 \cdot 3 \cdot 7}$</td>
<td>$2 \cdot 3\sqrt{7}$</td>
<td>$6\sqrt{7}$</td>
<td>6</td>
<td>-</td>
<td>0</td>
</tr>
<tr>
<td>Mo</td>
<td>$\sqrt{450}$</td>
<td>$\sqrt{5 \cdot 5 \cdot 3 \cdot 3 \cdot 2}$</td>
<td>$5 \cdot 3\sqrt{2}$</td>
<td>$15\sqrt{2}$</td>
<td>15</td>
<td>-</td>
<td>15</td>
</tr>
<tr>
<td>Curly</td>
<td>$\sqrt{980}$</td>
<td>$\sqrt{7 \cdot 7 \cdot 5 \cdot 2 \cdot 2}$</td>
<td>$7 \cdot 2\sqrt{5}$</td>
<td>$14\sqrt{5}$</td>
<td>14</td>
<td>6</td>
<td>84</td>
</tr>
</tbody>
</table>

**GAME 1**

<table>
<thead>
<tr>
<th>Player Name</th>
<th>Original Card</th>
<th>Place Mat</th>
<th>“Go Factor”</th>
<th>Simplified</th>
<th>Points Earned</th>
<th>Points Stolen</th>
<th>Total Points</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
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</table>
### GAME 2

<table>
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<th>Player Name</th>
<th>Original Card</th>
<th>Place Mat</th>
<th>“Go Factor”</th>
<th>Simplified</th>
<th>Points Earned</th>
<th>Points Stolen</th>
<th>Total Points</th>
</tr>
</thead>
<tbody>
<tr>
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</tbody>
</table>

### GAME 3

<table>
<thead>
<tr>
<th>Player Name</th>
<th>Original Card</th>
<th>Place Mat</th>
<th>“Go Factor”</th>
<th>Simplified</th>
<th>Points Earned</th>
<th>Points Stolen</th>
<th>Total Points</th>
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</thead>
<tbody>
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</tbody>
</table>
Simplifying Radicals, Day 2

HW

Quadratic Unit
Algebra 1

Simplify the radicals.

1. \(\sqrt{50}\)  2. \(3\sqrt{72} \cdot \sqrt{675}\)  3. \(\sqrt{81} \cdot -\sqrt{81}\)

4. \(\sqrt{9(2)}\)  5. \(-\sqrt{32}\)  6. \(\sqrt{162}\)

7. \(10 \cdot \sqrt{1000}\)  8. \(3\sqrt{9} \cdot -2\sqrt{36}\)  9. \(\sqrt{605}\)
Review
For each of the following problems identify what type of factoring is necessary. After, factor completely. (Remember, we have used factor by: Reverse Distributing, Grouping, Trinomials, and Difference of Squares)

10. $15x^2y - 10xy^2$ Type: _____________
11. $21 - 7x + 3y - xy$ Type: _____________

12. $x^2 - 8x + 16$ Type: _____________
13. $6x^2 - 11x - 2$ Type: _____________
To simplify a radical in rational form, multiply the numerator and denominator by the denominator radical. Simplify the fraction. The entire process is called rationalizing the denominator.

Example: \[
\frac{\sqrt{32}}{\sqrt{3}} = \frac{\sqrt{2 \cdot 2 \cdot 2 \cdot 2 \cdot 3}}{\sqrt{3}} = \frac{2 \cdot 2 \cdot \sqrt{2 \cdot 3}}{\sqrt{3}} = \frac{2 \cdot 2 \cdot \sqrt{2}}{\sqrt{3}} = \frac{4 \sqrt{6}}{3}
\]

1. \[
\frac{\sqrt{5}}{\sqrt{10}}
\]

2. \[
\frac{2\sqrt{27}}{\sqrt{6}}
\]

3. \[
\frac{\sqrt{9}}{\sqrt{3}}
\]

4. \[
\frac{\sqrt{18}}{\sqrt{64}}
\]

5. \[
\frac{-7}{2\sqrt{42}}
\]

6. \[
\frac{\sqrt{24}}{3}
\]
Simplifying Radicals, Day 3
HW
Quadratic Unit
Algebra 1

Simplify the radicals (rationalize the denominator).

1. \( \sqrt{\frac{128}{2}} \)
2. \( \frac{\sqrt{3}}{\sqrt{20}} \)
3. \( \frac{\sqrt{24}}{2} \)

4. \( \frac{4\sqrt{64}}{2\sqrt{25}} \)
5. \( \frac{\sqrt{5}}{2\sqrt{3}} \)
6. \( -\frac{5\sqrt{7}}{\sqrt{50}} \)
Review
For each of the following problems identify what type of factoring is necessary. After, factor completely. (Remember, we have used factor by: Reverse Distributing, Grouping, Trinomials, and Difference of Squares)

7. $2x^3y - x^2y + 5xy^2 + xy^3$ Type: __________
8. $2x^2 - 11x - 21$ Type: ________________

9. $x^2 - 2xy + x - 2y$ Type: ________________
10. $9x^2 - 64$ Type: ________________
Example 1:

a. Graph $y = x - 3$ on the grid and in $Y_1$.

b. Graph $y = x + 1$ on the grid and in $Y_2$.

c. Complete the $y$-values in the table below algebraically or using TABLE. Then use the $y$-values you found to complete the bottom row of the table.

<table>
<thead>
<tr>
<th>x-values</th>
<th>-2</th>
<th>-1</th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
</tr>
</thead>
<tbody>
<tr>
<td>y-values for $(x - 3)$</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>y-values for $(x + 1)$</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Multiply y-values for $(x - 3)$ and $(x - 1)$ together</td>
<td></td>
<td></td>
<td></td>
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<td></td>
<td></td>
</tr>
</tbody>
</table>

d. Make a list of coordinates where $x =$ the $x$-value from the table and $y =$ the multiplied value you found. Plot and connect these points on the grid above.

e. Write the vertex form of the quadratic function.

f. Change the vertex form of the function to standard form.

g. Double distribute the two linear functions. What do you notice about this equation and the standard form equation?
h. Find the x-intercepts of the linear functions (y = 0).

i. Where does the quadratic equation have x-intercepts (solutions)? Relate these to the linear functions.

Example 2:

a. Graph \( y = -x - 4 \) on the grid and in \( Y_1 \).

b. Graph \( y = x - 2 \) on the grid and in \( Y_2 \).

c. Complete the y-values in the table below algebraically or using TABLE. Then use the y-values you found to complete the bottom row of the table.

<table>
<thead>
<tr>
<th>x-values</th>
<th>(-4)</th>
<th>(-3)</th>
<th>(-2)</th>
<th>(-1)</th>
<th>(0)</th>
<th>(1)</th>
<th>(2)</th>
</tr>
</thead>
<tbody>
<tr>
<td>y-values for ((-x - 4))</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>y-values for ((x - 2))</td>
<td></td>
<td></td>
<td></td>
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<td></td>
<td></td>
</tr>
<tr>
<td>Multiply y-values for ((-x - 4)) and ((x - 2)) together</td>
<td></td>
<td></td>
<td></td>
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</tr>
</tbody>
</table>

d. Make a list of coordinates where \( x = \) the x-value from the table and \( y = \) the multiplied value you found. Plot and connect these points on the grid above.

e. Write the vertex form of the quadratic function.
f. Change the vertex form of the function to standard form.

g. Double distribute the two linear functions. What do you notice about this equation and the standard form equation?

h. Find the x-intercepts of the linear functions (y = 0).

i. Where does the quadratic equation have x-intercepts (solutions)? Relate these to the linear functions.

j. Where is the vertex of the quadratic function? What type of critical point is it?

k. Draw the line of symmetry on the graph with a dotted line. What is the equation of the LOS?

l. Give the y-intercept of the quadratic function.

m. What is the domain and range of the quadratic function?

n. Describe the behavior of the quadratic graph. Is it increasing? Decreasing? Write the intervals for each.
Multiplying Linear Functions

Name___________________________

Hour_______ Date ________________

Algebra 1

1.
   a. Graph \( y = x + 3 \) on the grid and in \( Y_1 \).

   b. Graph \( y = x + 7 \) on the grid and in \( Y_2 \).

   c. Complete the \( y \)-values in the table below algebraically or using TABLE. Then use the \( y \)-values you found to complete the bottom row of the table.

<table>
<thead>
<tr>
<th>( x )-values</th>
<th>-8</th>
<th>-7</th>
<th>-6</th>
<th>-5</th>
<th>-4</th>
<th>-3</th>
<th>-2</th>
</tr>
</thead>
<tbody>
<tr>
<td>( y )-values for ( x + 3 )</td>
<td></td>
<td></td>
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<td></td>
<td></td>
</tr>
<tr>
<td>( y )-values for ( x + 7 )</td>
<td></td>
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<td></td>
<td></td>
</tr>
<tr>
<td>Multiply ( y )-values for ( x + 3 ) and ( x + 7 ) together</td>
<td></td>
<td></td>
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</tbody>
</table>

d. Make a list of coordinates where \( x = \) the \( x \)-value from the table and \( y = \) the multiplied value you found. Plot and connect these points on the grid above.

e. Write the vertex form of the quadratic function.

f. Change the vertex form of the function to standard form.

g. Double distribute the two linear functions. What do you notice about this equation and the standard form equation?
h. Find the x-intercepts of the linear functions (y = 0).

i. Where does the quadratic equation have x-intercepts (solutions)? Relate these to the linear functions.

j. Where is the vertex of the quadratic function? What type of critical point is it?

k. Draw the line of symmetry on the graph with a dotted line. What is the equation of the LOS?

l. Give the y-intercept of the quadratic function.

m. What is the domain and range of the quadratic function?

n. Describe the behavior of the quadratic graph. Is it increasing? Decreasing? Write the intervals for each.

2. 

a. Graph \( y = x - 9 \) on the grid and in \( Y_1 \).

b. Graph \( y = x - 5 \) on the grid and in \( Y_2 \).

c. Complete the \( y \)-values in the table below algebraically or using TABLE. Then use the \( y \)-values you found to complete the bottom row of the table.

<table>
<thead>
<tr>
<th>x-values</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
<th>9</th>
<th>10</th>
</tr>
</thead>
<tbody>
<tr>
<td>( y )-values for ( x - 9 )</td>
<td></td>
<td></td>
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<td></td>
</tr>
<tr>
<td>( y )-values for ( x - 5 )</td>
<td></td>
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<td></td>
</tr>
<tr>
<td>Multiply ( y )-values for ( x - 9 ) and ( x - 5 ) together</td>
<td></td>
<td></td>
<td></td>
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</tbody>
</table>
d. Make a list of coordinates where \( x = \) the x-value from the table and \( y = \) the multiplied value you found. Plot and connect these points on the grid above.

e. Write the vertex form of the quadratic function.

f. Change the vertex form of the function to standard form.

g. Double distribute the two linear functions. What do you notice about this equation and the standard form equation?

h. Find the x-intercepts of the linear functions (\( y = 0 \)).

i. Where does the quadratic equation have x-intercepts (solutions)? Relate these to the linear functions.

j. Where is the vertex of the quadratic function? What type of critical point is it?

k. Draw the line of symmetry on the graph with a dotted line. What is the equation of the LOS?

l. Give the y-intercept of the quadratic function.

m. What is the domain and range of the quadratic function?

n. Describe the behavior of the quadratic graph. Is it increasing? Decreasing? Write the intervals for each.
Romeo High School wants to put on a production of *Taming of the Shrew* this spring. They would like to at least cover the costs of putting on the show, and, ideally, they would like to make a profit so that they can put on a more elaborate show in the spring.

The students involved decide to do a revenue-cost-profit analysis. To compute the money the play will take in (revenue or $R$), they must multiply the number of people (attendance or $A$), and the amount they charge per person, (ticket price or $T$); so, $R(x) = A(x) \cdot T(x)$.

After looking over the receipts from the last few plays, they notice that the more they charge, the fewer people come. They conduct a poll of several classes and estimate that they will lose 20 people for every $.25 they raise the price. They know from past experience that the show will attract 1200 people if they charge $4 for a ticket. (For our assignment we will let all the people be children.)

1. Let $x$ represent the number of times the price is changed
   - Let $A$ represent the attendance
   - Let $T$ represent the ticket price
   - Let $R$ represent the revenue for a given value of $x$

   Fill in the table showing attendance, ticket price, and revenue for $x$ values from -4 to 4.

<table>
<thead>
<tr>
<th>$x$</th>
<th>-4</th>
<th>-3</th>
<th>-2</th>
<th>-1</th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
</tr>
</thead>
<tbody>
<tr>
<td>Attendance ($A$)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>1200</td>
<td></td>
<td></td>
<td></td>
<td></td>
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<tr>
<td>Ticket Price ($T$)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>$4$</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Revenue ($R$)</td>
<td></td>
<td></td>
<td></td>
<td></td>
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<td></td>
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<td></td>
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</tbody>
</table>
2. Write a rule for the attendance versus number of price changes \((A \text{ in terms of } x)\).

3. Write a rule for the ticket price versus number of price changes \((T \text{ in terms of } x)\).

4. Write a rule for the revenue versus number of price changes \((R \text{ in terms of } x)\). Remember that \(R(x) = A(x) \cdot T(x)\). Simplify into standard form.

5. Now use your table and rules to find the best price to charge. Remember that \(x\) represents the number of price changes that occur, not dollars or people.
   
   A. What is the best possible revenue?
   
   B. What should the ticket price be?
   
   C. How many people will come?

6. Explain how you arrived at your answers to #5.
There is a buzz in the air as the Tigers get ready to begin the new season. Mike Illitch wants to maximize his profits this season so that he can keep bringing in players to help the team win the World Series.

Mr. Illitch decides to focus on the infield box seats to maximize the ticket sales. Last season the infield box seats cost $38 per game. Mr. Illitch decides to survey the season ticket holders and finds that they will lose 30 people for every $0.75 they raise the price. They know from last year’s sales they will sell 4,700 tickets when charging $38 per game.

1. 
   - Let \( x \) represent the number of times the price is changed
   - Let \( A \) represent the attendance
   - Let \( T \) represent the ticket price
   - Let \( R \) represent the revenue for a given value of \( x \)

Fill in the table showing attendance, ticket price, and revenue for \( x \) values from -4 to 4.

<table>
<thead>
<tr>
<th>( x )</th>
<th>-4</th>
<th>-3</th>
<th>-2</th>
<th>-1</th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
</tr>
</thead>
<tbody>
<tr>
<td>Attendance (( A ))</td>
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<td></td>
<td></td>
<td></td>
<td>4,700</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Ticket Price (( T ))</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>$38</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Revenue (( R ))</td>
<td></td>
<td></td>
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<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Adapted from Holt High School Mathematics Department
2. Write a rule for the attendance versus number of price changes \((A \text{ in terms of } x)\).

3. Write a rule for the ticket price versus number of price changes \((T \text{ in terms of } x)\).

4. Write a rule for the revenue versus number of price changes \((R \text{ in terms of } x)\). Remember that \(R(x) = A(x) \cdot T(x)\). Simplify into standard form.

5. Now use your table and rules to find the best price to charge. Remember that \(x\) represents the number of price changes that occur, not dollars or people.
   
   A. What is the best possible revenue?

   B. What should the ticket price be?

   C. How many people will come?
Using your graphing calculator, solve by graphing. Round to the nearest hundredth.

Example: \( f(x) = 3x^2 + 9x \)

I. Press \( y= \)

II. Type in the Equation

III. \( 2^{\text{nd}} \rightarrow \text{Calc} \rightarrow \text{Zero} \)

IV. Choose a zero to find and give it a left and right bound.

V. Re-enter \( 2^{\text{nd}} \rightarrow \text{Calc} \rightarrow \text{Zero} \). Choose the other zero to find giving the left and right bound to it.

The solutions are \( x = -3 \) and \( x = 0 \). Using the same function, solve by factoring. Use factoring by reverse distribution. Set each factored term equal to 0 and solve for \( x \).

\[
\begin{align*}
f(x) &= 3x^2 + 9x \\
f(x) &= 3x(x + 3) \\
3x &= 0 & \text{and} & \ x + 3 &= 0 \\
\frac{3}{3} &= 0 & -3 &= -3 \\
x &= 0 & \ x &= -3
\end{align*}
\]

Compare these solutions with the solutions found by graphing. What can be concluded?

Solve the following function by graphing. Round to the nearest hundredth if necessary.

1. \( f(x) = -5x^2 - 6x + 17 \)
Solve the following functions by graphing and factoring. Write the type of factoring used.
2. \( f(x) = -14x^2 - 7x \) Type: _______________________

3. \( f(x) = x^2 - 16x + 64 \) Type: _______________________

4. \( f(x) = x^2 - 8x - 9 \) Type: _______________________

5. \( f(x) = 2x^2 + 11x + 12 \) Type: _______________________

6. \( f(x) = x^2 + 4x - 12 \) Type: _______________________

7. \( f(x) = 4x^2 - 81 \) Type: _______________________

Factored Form, Day 1

HW
Quadratic Unit
Algebra 1

Using your graphing calculator, solve by graphing. Round to the nearest hundredth if necessary.
1. \( f(x) = 6x^2 - 13x - 5 \)

2. \( f(x) = 3x^2 + 7x - 16 \)

3. \( f(x) = 20x^2 + 13x - 483 \)

Solve by factoring.
4. \( f(x) = x^2 - x - 20 \)

5. \( f(x) = x^2 - 15x \)

6. \( f(x) = x^2 + 8x + 7 \)

7. \( f(x) = 2x^2 - 3x - 20 \)
Review
Determine if each function is Linear, Quadratic or Neither. If it is Linear or Quadratic, give the equation.

8. | Δx | 1st difference | 2nd difference |
   | -3 |       |       |
   | -2 |       |       |
   | -1 |       |       |
   | 0  |       |       |
   | 1  |       |       |
   | 2  |       |       |

Test Practice
9. Simplify the radical expression: \(\sqrt{396}\)

a. \(2\sqrt{99}\)  
b. \(12\sqrt{3}\)  
c. \(6\sqrt{11}\)  
d. \(12\sqrt{6}\)

10. Which selection contains a factor of: \(24a^2b - 18ab^2\)?

a. \(24a^2b - 18ab^2\)  
b. \(2ab\)  
c. \(6ab\)  
d. \(6a^2b^2\)

11. Which selection contains a factor of: \(x^2 + 6x + 9\)?

a. \((x + 3)\)  
b. \((x - 3)\)  
c. \((x + 6)\)  
d. \((x + 9)\)
Given solutions, find a quadratic equation in standard form (no fractions).

1. (6, 0), (5, 0) and the quadratic function opens up

2. (-7, 0), (2, 0) and the quadratic function opens down

3. (-3, 0), (5/2, 0) and the quadratic function opens down

4. (3, 0) and the quadratic function opens up
Given a table of values, find a quadratic equation in factored form that fits the data points. Then, convert to express in standard form (assume that the quadratic function opens up):

<table>
<thead>
<tr>
<th></th>
<th>x</th>
<th>y</th>
</tr>
</thead>
<tbody>
<tr>
<td>5</td>
<td>-4</td>
<td>0</td>
</tr>
<tr>
<td></td>
<td>-3</td>
<td>-5</td>
</tr>
<tr>
<td></td>
<td>-1</td>
<td>-9</td>
</tr>
<tr>
<td></td>
<td>0</td>
<td>-8</td>
</tr>
<tr>
<td></td>
<td>2</td>
<td>0</td>
</tr>
<tr>
<td></td>
<td>3</td>
<td>7</td>
</tr>
<tr>
<td>6</td>
<td>-7</td>
<td>16</td>
</tr>
<tr>
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<td>-5</td>
<td>0</td>
</tr>
<tr>
<td></td>
<td>-2</td>
<td>-9</td>
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<td>0</td>
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<td></td>
<td>3</td>
<td>16</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th></th>
<th>x</th>
<th>y</th>
</tr>
</thead>
<tbody>
<tr>
<td>7</td>
<td>-8</td>
<td>0</td>
</tr>
<tr>
<td></td>
<td>-6</td>
<td>-4</td>
</tr>
<tr>
<td></td>
<td>-5</td>
<td>-3</td>
</tr>
<tr>
<td></td>
<td>-4</td>
<td>0</td>
</tr>
<tr>
<td></td>
<td>-1</td>
<td>21</td>
</tr>
<tr>
<td></td>
<td>0</td>
<td>32</td>
</tr>
<tr>
<td>8</td>
<td>-2</td>
<td>12</td>
</tr>
<tr>
<td></td>
<td>-1</td>
<td>6</td>
</tr>
<tr>
<td></td>
<td>0</td>
<td>2</td>
</tr>
<tr>
<td></td>
<td>1</td>
<td>0</td>
</tr>
<tr>
<td></td>
<td>2</td>
<td>0</td>
</tr>
<tr>
<td></td>
<td>3</td>
<td>2</td>
</tr>
</tbody>
</table>
Given solutions, find a quadratic equation in standard form (no fractions).

1. (9, 0), (1, 0) and the quadratic function opens down

2. (-7, 0), (-3, 0) and the quadratic function opens up

3. (-4, 0) and the quadratic function opens down

4. (-8, 0), (2/3, 0) and the quadratic function opens up
Given a table of values, find a quadratic equation in factored form that fits the data points. Then, convert to express in standard form (assume that the quadratic function opens up):

5. \[
\begin{array}{|c|c|}
\hline
x & y \\
\hline
0 & 12 \\
1 & 5 \\
2 & 0 \\
3 & -3 \\
6 & 0 \\
8 & 12 \\
\hline
\end{array}
\]

6. \[
\begin{array}{|c|c|}
\hline
x & y \\
\hline
-8 & 0 \\
-6 & -6 \\
-3 & 0 \\
0 & 24 \\
1 & 36 \\
6 & 126 \\
\hline
\end{array}
\]

7. \[
\begin{array}{|c|c|}
\hline
x & y \\
\hline
-10 & 0 \\
-5 & -30 \\
-1 & -18 \\
0 & -10 \\
1 & 0 \\
5 & 60 \\
\hline
\end{array}
\]

8. \[
\begin{array}{|c|c|}
\hline
x & y \\
\hline
-7 & 0 \\
-4 & -33 \\
0 & -49 \\
4 & -33 \\
7 & 0 \\
11 & 72 \\
\hline
\end{array}
\]
Completing the Square, Day 1

Notes

Quadratic Unit

Algebra 1

Review:
Standard Form: ____________________________ What can standard form tell us about our
graph? ______________________________________________________________________

Vertex Form: ____________________________ What can vertex form tell us about our
graph? ______________________________________________________________________

“Completing the Square” allows us to convert a quadratic expression or equation from
standard form to vertex form.

The Process

When given a quadratic in standard form: \( ax^2 + bx + c \),
1. Group the first two terms in parentheses.
2. Take \( b \) and divide it by 2.
3. Square the result from the previous step.
4. Add the result to \( ax^2 + bx \)
5. **Keep the equation balanced by subtracting what you added**
at the end of you expression.**
6. Factor the quantity in the parentheses and simplify the constant.

Example:
\[
\begin{align*}
\text{Example: } & \quad x^2 - 8x + 7 \\
& \quad = (x^2 - 8x) + 7 \\
& \quad \text{side note: } -8/2 = -4 \\
& \quad = (x^2 - 8x + 16) + 7 \\
& \quad = (x - 4)^2 + 7 - 16 \\
& \quad = (x - 4)(x - 4) - 9 \\
& \quad = (x - 4)^2 - 9
\end{align*}
\]

Find the value for “\( c \)” that makes the quadratic a perfect square trinomial, then factor it.

1. \( x^2 - 24x + c \)
2. \( x^2 + 46x + c \)
3. \( x^2 + 110x + c \)
4. \( x^2 - 38x + c \)
For each quadratic,
   a. Write in vertex form
   b. Give the vertex
   c. Find the equation for the AOS
   d. Find the y-intercept

5. \( y = x^2 + 10x + 5 \)

5a. _____________________
5b. _____________________
5c. _____________________
5d. _____________________

6. \( y = x^2 + 6x - 19 \)

6a. _____________________
6b. _____________________
6c. _____________________
6d. _____________________

7. \( y = x^2 - 12x + 21 \)

7a. _____________________
7b. _____________________
7c. _____________________
7d. _____________________
Completing the Square, Day 1
HW
Quadratic Unit
Algebra 1

For each quadratic,

a. Write in vertex form  
b. Give the vertex  
c. Find the equation for the AOS  
d. Find the y-intercept  

1. \(y = x^2 + 2x + 17\)

1a. _____________________
1b. _____________________
1c. _____________________
1d. _____________________

2. \(y = x^2 + 12x + 29\)

2a. _____________________
2b. _____________________
2c. _____________________
2d. _____________________

3. \(y = x^2 - 8x - 23\)

3a. _____________________
3b. _____________________
3c. _____________________
3d. _____________________
4. \( y = x^2 - 10x + 7 \)

5. \( y = x^2 - 4x - 13 \)

Test Practice

6. Given the following table, find the correct “b” value and identify the type of function.

<table>
<thead>
<tr>
<th>( \Delta x )</th>
<th>( x )</th>
<th>( y )</th>
<th>1st difference</th>
<th>2nd difference</th>
</tr>
</thead>
<tbody>
<tr>
<td>-5</td>
<td>-91</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>-4</td>
<td>-68</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>-3</td>
<td>-49</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>-2</td>
<td>-34</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>-1</td>
<td>-23</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>0</td>
<td>-16</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

a. Quad, b = 5     b. Linear, b = -16     c. Linear, b = -91     d. Quad, b = 10
Completing the Square, Day 2

Notes
Quadratic Unit
Algebra 1

The examples today follow the same process as “Completing the Square #1”, but with these examples, some extra steps are required before the process can begin.

Convert the following quadratic equations into vertex form. Give the vertex and the AOS.

Ex: \( x^2 + \frac{3}{2}x + \frac{1}{2} = y \)

**Finding the new “c” value:**

\[
\frac{3}{2} \div 2 = \frac{3}{4} \\
\downarrow \\
\left( \frac{3}{4} \right)^2 = \frac{9}{16}
\]

- Take “b” and divide it by 2
- Square that result

**Completing the Square:**

\[
\left( x^2 + \frac{3}{2}x + \frac{9}{16} \right) + \frac{1}{2} - \frac{9}{16} = y \\
\left( x + \frac{3}{4} \right) \cdot \left( x + \frac{3}{4} \right) - \frac{1}{16} = y \\
\left( x + \frac{3}{4} \right)^2 - \frac{1}{16} = y \\
y = \left( x + \frac{3}{4} \right)^2 - \frac{1}{16}
\]

- Add \( \frac{9}{16} \) inside the parenthesis and subtract it outside
- Factor the quadratic and simplify the outside numbers
- Simplify
- Vertex located at \( \left( -\frac{3}{4}, -\frac{1}{16} \right) \)
- Axis of Symmetry (AOS, a.k.a. line of symmetry) located at \( x = -\frac{3}{4} \)
1. \( x^2 + 3x - 18 = y \)  
2. \( x^2 - \frac{5}{3}x + \frac{2}{3} = y \)  

3. \( x^2 + 5x - 24 = y \)  
4. \( x^2 + \frac{23}{5}x + \frac{12}{5} = y \)
Completing the Square, Day 2

HW
Quadratic Unit
Algebra 1

Convert the following quadratic equations into vertex form. Give the vertex and the AOS.

1. \( x^2 + 5x - 6 = y \)

2. \( x^2 - \frac{19}{4}x + 3 = y \)

3. \( x^2 + \frac{7}{4}x - \frac{1}{2} = y \)

4. \( x^2 - 10x + 16 = y \)
Convert to vertex form, and then graph each quadratic equation. Also, find and label the solutions on the graph.

5.  \( y = x^2 - 4x - 1 \)

6.  \( f(x) = x^2 - 6x + 5 \)

*Test Practice*

7. Solve the quadratic equation by factoring:  \( f(x) = x^2 + 8x + 12 \)

a. Factors: \((x + 8)(x + 12)\)  
Solutions: \((-8,0)(-12,0)\)

b. Factors: \((x + 6)(x + 2)\)  
Solutions: \((6,0)(2,0)\)

c. Factors: \((x - 6)(x - 2)\)  
Solutions: \((6,0)(2,0)\)

d. Factors: \((x + 6)(x + 2)\)  
Solutions: \((-6,0)(-2,0)\)
Solving Using Square Roots, Day 1

Notes
Quadratic Unit
Algebra 1

Simplify.

1. $-3 \times -3$  
2. $3 \times 3$  
3. $-4 \times -4$  
4. $4 \times 4$

What can be concluded?

Up until now, we have always solved multi-step equations using a linear equation. Now, we are going to solve for $x$ using a quadratic equation.

Solve for $x$ and simplify all radicals.

**EXAMPLE:**

$$x^2 - 4 = 5$$

$$x^2 = 9$$

$$\sqrt{x^2} = \sqrt{9}$$

$$x = \pm 3$$

- Isolate the squared term by adding 4 to both sides
- Simplify
- To undo squaring...square root both sides
- Always account for the ± (per our examples above)

5. $(x - 3)^2 = 16$

6. $2x^2 = 196$
7. \(6(x + 6)^2 + 14 = 878\)

8. \(4x^2 - 25 = 0\)

For each quadratic,

a. Write in vertex form
b. Give the vertex
c. Find the equation for the AOS
d. Find the x-intercepts
e. Write in factored form

**EXAMPLE:** \(y = x^2 - 4x\)

a. \(\frac{-4}{2} = -2, (-2)^2 = 4\)

b. \(\sqrt{4} = \sqrt{(x-2)^2}\)

c. \(\pm 2 = x - 2\)

d. \(0 = (x - 2)^2 - 4\)

e. \((4, 0)\) \((0, 0)\)
9. \( y = x^2 - 2x - 15 \) 

10. \( y = x^2 - 10x - 24 \) 

11. \( y = x^2 + 4x - 5 \)
Solve for $x$ and simplify radicals.

1. $4x^2 - 7 = 21$
2. $2(x + 5)^2 - 4 = 196$
3. $2x^2 + 9 = 19$
4. $(x - 3)^2 + 1 = 17$
5. $3(x - 2)^2 = 48$
6. $3(x - 7)^2 + 6 = 249$
For each quadratic,
   a. Write in vertex form
   b. Give the vertex
   c. Find the equation for the AOS
   d. Find the x-intercepts
   e. Write in factored form
   f. Give the y-intercept

7. \( y = x^2 - 12x + 27 \)
   a. 
   b. 
   c. 
   d. 
   e. 
   f. 

8. \( y = x^2 - 8x - 20 \)
   a. 
   b. 
   c. 
   d. 
   e. 
   f. 

9. \( y = x^2 + 6x - 16 \)
   a. 
   b. 
   c. 
   d. 
   e. 
   f. 
Completing the Square and Solving.

Follow the example below to help solve for $x$:

Example:

1. $15 = x^2 + 2x$
2. $0 = x^2 + 2x - 15$
3. $0 = (x^2 + 2x + 1) - 15 - 1$
   "Complete the square"
4. $0 = (x + 1)^2 - 16$
   "Simplify to vertex form"

Now solve for $x$:
1. $16 = (x + 1)^2$
2. $\pm \sqrt{16} = x + 1$
3. $-1 \pm 4 = x$
4. $x = 3$ and $x = -5$

Solution(s): _____________________________

2. $x^2 - 6x + 8 = 0$

Solution(s): _____________________________
Solve using square roots.

3. \( \frac{x^2 + 5}{3} = 122 \)

Solution(s):

4. \( \frac{x^2}{6} - 24 = 0 \)

Solution(s):
Solve for $x$ and simplify radicals.

1. \[ \frac{2(x + 7)^2}{3} = 54 \]
2. \[ \frac{(x - 8)^2}{5} = 45 \]

3. \[ 2x^2 + 15 = 231 \]
4. \[ \frac{3x^2 - 750}{25} = 66 \]

5. \[ 4(x - 15)^2 - 27 = 229 \]
6. \[ 2(x - 3)^2 = 50 \]
7. \(3x^2 + 5 = 32\)  
8. \((x + 4)^2 - 5 = 20\)

Solve the following quadratics by completing the square.

9. \(-20x = x^2 + 99\)

10. \(0 = x^2 - 14x + 33\)

11. \(x^2 = -16x - 60\)
12. \( 0 = x^2 - 6x - 16 \)

**Test Practice**

13. Given the solutions of \((4, 0)\) and \((-2, 0)\), identify the quadratic equation in standard form given the parabola opens up.

a. \( y = x^2 - 2x - 8 \)  
   b. \( y = -x^2 + 2x + 8 \)  
   c. \( y = -x^2 - 2x + 8 \)  
   d. \( y = x^2 + 2x - 8 \)

14. The parabola opens down and has one solution at \((-6, 0)\). Which of the following equations does not represent the given quadratic function?

a. \( y = -(x + 6)^2 \)  
   b. \( y = -x^2 - 12x - 36 \)  
   c. \( y = -(x + 6)(x + 6) \)  
   d. \( y = x^2 + 12x - 36 \)
Imaginary Numbers
Notes
Quadratic Unit
Algebra 1

- We’ve always said that it is impossible to take the square root of a negative number.
- This is true when we’re dealing with real numbers.
- However, the square root of a negative number can be expressed as “i” when using imaginary numbers.

EXAMPLE: Simplify $\sqrt{-9}$.

- With real numbers, we would say there is no solution.
- With imaginary numbers, we would say the solution is $3i$.

Since $i = \sqrt{-1}$, we can also say that $i^2 = \sqrt{-1} \cdot \sqrt{-1} = -1$

- Whenever there is a negative number inside a square root, immediately pull it out front and call it “i”.
- Imaginary numbers are used to represent quadratic functions that have no x-intercepts (i.e. there is no solution).

Simplify each problem below.
Example: $\sqrt{-128}$

\[
\begin{align*}
\sqrt{-128} &= \sqrt{-1} \cdot \sqrt{128} \\
&= i \cdot \sqrt{2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2} \\
&= 2 \cdot 2 \cdot 2 \cdot i \sqrt{2} \\
&= 8i \sqrt{2}
\end{align*}
\]
2. $\sqrt{-243}$  
3. $7\sqrt{-144}$

EXAMPLE: Simplify $\sqrt{-42} \cdot \sqrt{-6}$

$\sqrt{-1} \cdot \sqrt{42} \cdot \sqrt{-1} \cdot \sqrt{6}$

$i \cdot i \cdot \sqrt{42} \cdot \sqrt{6}$

$i^2 \sqrt{252}$

$i^2 \sqrt{2 \cdot 2 \cdot 3 \cdot 3 \cdot 7}$

$2 \cdot 3 \cdot -1\sqrt{7}$

$-6\sqrt{7}$

Simplify each problem below.

4. $\sqrt{-400} \cdot \sqrt{90}$

5. $-3\sqrt{-5} \cdot 4\sqrt{-20}$
We can also use these new ideas to find imaginary solutions to equations that normally have no real solutions.

**EXAMPLE:** \[ x^2 + 16 = 0 \]

\[
\Rightarrow x^2 = -16 \quad - \text{Subtract 16 from both sides}
\]

\[
\Rightarrow \sqrt{x^2} = \sqrt{-16} \quad - \text{Square root both sides to “un-do” the square}
\]

\[
\Rightarrow x = \pm 4i \quad - \text{Simplify } \sqrt{-16}
\]

Solve each equation below.

6. \(5x^2 = -125\)  
7. \(2r^2 + 64 = 0\)

8. \(20x^2 = 4x^2 - 320\)  
9. \(-4(2k^2 - 1) = 14\)
Simplify each radical.

1. \( \sqrt{-512} \)  
2. \( 3\sqrt{-12} \cdot \sqrt{6} \)

3. \( \sqrt{-80} \)  
4. \( -4\sqrt{15} \cdot -\sqrt{3} \)

5. \( \sqrt{75} \)  
6. \( \sqrt{-5} \cdot \sqrt{-5} \)

7. \( \sqrt{-96} \)  
8. \( -4\sqrt{-28} \cdot -3\sqrt{-7} \)
Solve for $x$, simplifying radicals when necessary.

9. $x^2 = 76$  
10. $28 = x^2 + 8$

11. $x^2 = -21$  
12. $-6x^2 = -384$

13. $10 = 5x^2 + 35$  
14. $x^2 = 2x^2 + 24$

15. $6x^2 + 48 = 0$  
16. $4(x^2 - 3) = x^2 - 10$
Another method to solve for $x$ in a quadratic equation is to use the quadratic formula. The quadratic formula can be used to solve for $x$ when the form $ax^2 + bx + c = 0$ where $a \neq 0$.

The quadratic formula is:

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

**The Discriminant: $b^2 - 4ac$**

- If $b^2 - 4ac > 0$, then we have ________ real solutions.
- If $b^2 - 4ac = 0$, then we have ________ solution.
- If $b^2 - 4ac < 0$, then we have ________ real solutions. This will produce __________ solutions.

Example: $f(x) = 2x^2 - 5x - 3$

a. Solve using the quadratic formula.

$$x = \frac{-(-5) \pm \sqrt{(-5)^2 - 4(2)(-3)}}{2(2)}$$

$$x = \frac{5 \pm \sqrt{25 + 24}}{4}$$

$$x = \frac{5 \pm \sqrt{49}}{4}$$

$$x = \frac{5 \pm 7}{4}$$

$$x = \frac{5 + 7}{4} \text{ and } x = \frac{5 - 7}{4}$$

$$x = \frac{5}{4} \text{ and } x = \frac{-1}{2}$$

b. Tell how many solutions there are.

**Two Real Solutions**

c. Graph the quadratic and confirm the Real solutions are at 3 and $-\frac{1}{2}$.
1. \( f(x) = -x^2 - 12x - 32 \)
   a. Solve using the quadratic formula.
   b. Tell how many solutions there are.
   c. Graph the quadratic to confirm the solutions.

2. \( f(x) = -x^2 - 4x - 5 \)
   a. Solve using the quadratic formula.
   b. Tell how many solutions there are.
   c. Graph the quadratic to confirm the solutions.
3. \( f(x) = 4x^2 + 4x + 1 \)
   
   a. Solve using the quadratic formula.

   b. Tell how many solutions there are.

   c. Graph the quadratic to confirm the solutions.

4. \( f(x) = 2x^2 - 7x + 1 \)

   a. Solve using the quadratic formula.

   b. Tell how many solutions there are.

   c. Graph the quadratic to confirm the solutions.
Solve each quadratic by factoring, graphing, completing the square, and the quadratic formula. Parts a., b., c., and d., should all produce the same solutions.

1. $x^2 - 8x + 7 = 0$
   a. Factoring
   b. Graphing
   c. Completing the Square
   d. Quadratic Formula
2. \( x^2 + 10x + 24 = 0 \)
   
   a. Factoring  
   b. Graphing
   
   c. Completing the Square
   
   d. Quadratic Formula
Solve each quadratic by factoring, graphing, completing the square, and the quadratic formula. Remember, all four methods should produce the same solutions.

1. \(x^2 + 14x + 40 = 0\)
   - a. Factoring
   - b. Graphing
   - c. Completing the Square
   - d. Quadratic Formula
2. \( x^2 - 10x + 16 = 0 \)
   
   a. Factoring
   b. Graphing
   c. Completing the Square
   d. Quadratic Formula
3. $x^2 - 2x - 8 = 0$
   a. Factoring
   b. Graphing
   c. Completing the Square
   d. Quadratic Formula
4. $x^2 - 10x + 24 = 0$

a. Factoring  

b. Graphing


c. Completing the Square

d. Quadratic Formula
Just as we solved a system of equations in the Linear Unit, we are going to solve a system of equations with quadratic functions. The solution to a system of equations represents the __________________________ of the two parabolas.

There are either 0, 1, 2, or infinitely many intersection points between the two quadratic functions.

Example: Solve the following system of equations by graphing.

\[
\begin{align*}
7 & - x^2 + 1 \\
2 & + x^2 - 7
\end{align*}
\]

The 1st function is flipped, shifted left 1 and up 1. The 2nd function is shifted left 1 and down 7.

The two parabolas intersect at (-3, -3) and (1, -3).

Let’s show what the intersection point represents in regards to the system of equations.

Solve the following system of equations by graphing.

1. \[
\begin{align*}
f(x) & = (x - 4)^2 - 3 \\
f(x) & = (x - 7)^2
\end{align*}
\]
2. \(f(x) = -(x+2)^2 - 6\)
   \(f(x) = (x+7)(x+3)\)

3. \(f(x) = (x-8)(x-4)\)
   \(f(x) = (x-6)^2 - 4\)

4. \(f(x) = -(x-5)^2 + 7\)
   \(f(x) = (x-7)^2 + 3\)
Estimate the solutions of each system of equations graphed.

1. \( y = x^2 - 5 \)  
   \( y = -x^2 + 3 \)

2. \( y = -x^2 + 4x - 3 \)  
   \( y = x^2 - 1 \)

3. \( y = -x^2 + 3x \)  
   \( y = x^2 + x \)

Solve each system by graphing. Use a graphing calculator. Round coordinates to the nearest hundredth.

4. \( y = x^2 + 3x - 5 \)  
   \( y = -x^2 + 4 \)

5. \( y = 0.75x^2 - 2x + 1 \)  
   \( y = -0.5x^2 + 3x + 1 \)

6. \( y = -2x^2 - 3x + 4 \)  
   \( y = -x^2 - 2x + 2 \)

7. \( y = -0.4x^2 + x + 2 \)  
   \( y = -0.4x^2 + 2x + 1 \)

8. \( y = 2x^2 - 4x - 7 \)  
   \( y = -1.5x^2 + 2.5x - 1 \)

9. \( y = -2x^2 + 6x - 3 \)  
   \( y = x^2 - 6x + 9 \)
Solve the following system of equations by graphing.

10. \[ f(x) = x(x - 4) \]
    \[ f(x) = -(x - 2)^2 + 4 \]

11. \[ f(x) = (x - 2)^2 - 7 \]
    \[ f(x) = -(x + 5)^2 - 5 \]

12. \[ f(x) = -(x + 3)(x + 9) \]
    \[ f(x) = -(x + 3)^2 + 6 \]
Remember, to solve a quadratic system you are looking for the intersection points. Answers are written as coordinate points \((x, y)\). There are 0, 1, 2, or infinitely many intersection points between the two quadratic functions.

Example:
\[
\begin{align*}
y &= 2x^2 + 4 \\
y &= -3x^2 - 3
\end{align*}
\]

Solve by Substitution
\[
\begin{align*}
2x^2 + 4 &= -3x^2 - 3 \\
+3x^2 &= 0 \\
5x^2 + 4 &= -3 \\
-4 &= 0 \\
5x^2 &= -7 \\
x^2 &= \frac{-7}{5} \\
x &= \pm \sqrt{\frac{-7}{5}}
\end{align*}
\]

There are no intersection points because of the negative square root. Remember this produces imaginary numbers.

1. \[
\begin{align*}
y &= .25x^2 + 4 \\
y &= -.25x^2 + 6
\end{align*}
\]

Solve by Substitution or Elimination
2. \[ y = x^2 - 8x + 17 \]
\[ y = -x^2 + 8x - 15 \]

Solve by Substitution or Elimination

3. \[ y = x^2 + 2x + 2 \]
\[ y = -x^2 - 4x + 10 \]

Solve by Substitution or Elimination
Solve each system algebraically. Use your calculator to check your answers. Write final answers as coordinates.

1. \( y = x^2 + 6x + 6 \)
   \( y = -x^2 - 4x - 6 \)

2. \( y = -x^2 + 6x - 4 \)
   \( y = x^2 - 4 \)

3. \( y = -x^2 + 7 \)
   \( y = x^2 - 1 \)

4. \( y = -3x^2 + 2x + 4 \)
   \( y = -3x^2 \)
5. \( y = x^2 - 8x + 19 \) 
   \( y = -x^2 - 4x + 1 \)

6. \( y = x^2 + 10x + 23 \) 
   \( y = -x^2 - 14x - 47 \)

7. \( y = x^2 \) 
   \( y = x^2 - 8x + 8 \)

8. \( y = 4x^2 + 43 \) 
   \( y = x^2 - 2 \)
Quadratic Inequalities
Notes
Quadratic Unit
Algebra 1

With inequalities, shading is used to represent the solution.

The Process:

- Graph the quadratic function. (If the inequality is > or < the curve will be a dashed; If the inequality is ≤ or ≥ the curve will be solid.)
- Pick a test point either above or below the curve. If it satisfies the inequality shade towards the test point. If it doesn’t satisfy the inequality shade away on the other side of the curve.

Example: $x^2 + 4x - 3 > y$

Either factor or complete square to graph the quadratic.

Complete the Square:

\[
\frac{4}{2} = 2 \rightarrow (2)^2 = 4 \\
x^2 + 4x - 3 > y \\
(x^2 + 4x + 4) - 3 - 4 > y \\
(x + 2)^2 - 7 > y \\
\text{Vertex: (-2 , -7)} \\
\text{Graph it!}
\]

Test Point:

Choose (0 , 0)

\[0^2 + 4(0) - 3 > 0?\]

\[0 > 3?\]

False!

So shade on the side of the quad. that does not contain (0 , 0).

1. Solve $x^2 + 2x + 5 \leq y$
2. Solve \( x^2 - 12x + 36 > y \)

3. Solve \(- (x + 7)^2 \geq y\)

4. Solve \(4x^2 - 9x + 4 < y\)

Get a table of values to graph!

<table>
<thead>
<tr>
<th>x</th>
<th>y</th>
</tr>
</thead>
</table>
Solve each quadratic inequality.

1. $x^2 + 6x + 6 \geq y$

2. $y > x^2 - 10x + 21$

3. $x^2 + 4x - 1 \geq y$
4. \( x^2 - 2x + 6 < y \)

5. \( y > x^2 - 9 \)

6. \(-3x^2 + 7x + 5 \leq y \)

Get a table of values to graph!

<table>
<thead>
<tr>
<th>x</th>
<th>y</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
</tr>
</tbody>
</table>
1. The formula \( h = 200t - 2t^2 \) gives the height, \( h \), in meters, of a rocket \( t \) seconds after take-off. The maximum height reached by the rocket is 5000 m. How long will it take the rocket to reach this maximum height?

2. A rectangular garden was 16 m wide and 30 m long. The area of the garden was increased to 912 m\(^2\) by digging a uniform border around the garden. Find the width of the border.
3. The square of a number is 81 less than 18 times the number. Find the number.

4. The sum of two numbers is 32. The sum of the squares of two numbers is 544. Find the numbers.

5. A swimming pool 20 m long and 10 m wide is surrounded by a deck of uniform width. The total area of the swimming pool and the deck is 704m$^2$. Find the width of the deck.
6. A rectangle was 25 cm longer than it was wide. A new rectangle was formed by decreasing the length by 6 cm and decreasing the width by 5 cm. The area of the new rectangle was $585 \text{ cm}^2$. Find the dimensions of the original rectangle.

7. A clothing store sells 40 pairs of jeans daily at $30 each. The owner figures that for each $3 increase in price, 2 fewer pairs will be sold each day. What price should be charged to maximize profit?
8. An object is thrown upward into the air with an initial velocity of 128 feet per second. The formula \( h(t) = 128t - 16t^2 \) gives its height above the ground after \( t \) seconds. What is the height after 2 seconds? What is the maximum height reached? For how many seconds will the object be in the air?

9. Find the dimensions and maximum area of a rectangle if its perimeter is 48 inches.

10. A square, which is 2 inches by 2 inches, is cut from each corner of a rectangular piece of metal. The sides are folded up to make a box. If the bottom must have a perimeter of 32 inches, what would be the length and width for maximum volume?
Standard form for quadratic functions is written as \( y = ax^2 + bx + c \), where “a” can represent any real number.

We mostly have been working with quadratics that have an “a” value of 1, but we have seen other real numbers. In today’s lesson, we are going to find the equation of a quadratic function in standard form with an “a” value other than 1 (\( a \neq 1 \)). Pattern of points cannot be used when graphing these functions because “a” \( \neq 1 \).

Example:
Determine the quadratic function that contains the points (-3, 0), (6, 135) and (-6, 0).

Example:
Find the equation for a quadratic function with vertex (2, 6) and contains the point (6, -58).

Example:
7. Use the table of values to determine the equation of the quadratic function:

<table>
<thead>
<tr>
<th>( x )</th>
<th>( y )</th>
</tr>
</thead>
<tbody>
<tr>
<td>-6</td>
<td>-5</td>
</tr>
<tr>
<td>-4</td>
<td>0.5</td>
</tr>
<tr>
<td>-2</td>
<td>1.5</td>
</tr>
<tr>
<td>0</td>
<td>2.5</td>
</tr>
</tbody>
</table>

\( x = -6 \)
1. Determine the quadratic function that contains the points (-2, 0), (4, 0) and (2, 24).

2. Determine the quadratic function that contains the points (-2, -18), (-5, 0) and (1, 0).

3. Find the equation for a quadratic function with vertex (-7, -1) and contains the point (5, 95).
4. Find the equation for a quadratic function with vertex (-8, 3) and contains the point (-15, -25).

5. Use the table of values to determine the equation of the quadratic function:

<table>
<thead>
<tr>
<th>X</th>
<th>Y1</th>
</tr>
</thead>
<tbody>
<tr>
<td>-2.4</td>
<td>0</td>
</tr>
<tr>
<td>1.6</td>
<td>2.4</td>
</tr>
<tr>
<td>2.4</td>
<td>1.6</td>
</tr>
<tr>
<td>0</td>
<td>0</td>
</tr>
</tbody>
</table>

6. Use the table of values to determine the equation of the quadratic function:

<table>
<thead>
<tr>
<th>X</th>
<th>Y1</th>
</tr>
</thead>
<tbody>
<tr>
<td>-2.667</td>
<td>24</td>
</tr>
<tr>
<td>5.333</td>
<td>13.333</td>
</tr>
<tr>
<td>0</td>
<td>5.333</td>
</tr>
<tr>
<td>2.667</td>
<td>-2.667</td>
</tr>
<tr>
<td>0</td>
<td>0</td>
</tr>
</tbody>
</table>
1. \( y = (x - 3)^2 \)
   a. Graph and describe the translations.
   b. Give the type and coordinate of the vertex.
   c. Find the y-intercept.
   d. What is the range of the function?
   e. Draw the axis of symmetry with a dotted line. Give the equation of the axis of symmetry.

2. \( y = (x + 2)^2 + 1 \)
   a. Graph and describe the translations.
   b. Identify the solutions (also called zeroes or x-intercepts).
   c. Find the y-intercept.
   d. What is the domain of the function? What is the range of the function?
   e. Draw the axis of symmetry with a dotted line. Give the equation of the axis of symmetry.
3. \( y = (x - 4)^2 - 9 \)
   a. Graph and describe the translations.
   b. Give the type and coordinate of the vertex.
   c. Identify the solutions.
   d. Find the \( y \)-intercept.
   e. What is the domain of the function? What is the range of the function?
   f. Draw the axis of symmetry with a dotted line. Give the equation of the axis of symmetry.

4. \( y = -(x - 3)^2 - 4 \)
   a. Graph and describe the translations.
   b. Give the type and coordinate of the vertex.
   c. Identify the solutions.
   d. Find the \( y \)-intercept.
   e. What is the domain of the function? What is the range of the function?
   f. Draw the axis of symmetry with a dotted line. Give the equation of the axis of symmetry.
Example: Solve the piecewise function for each value given.

\[
f(x) = \begin{cases} 
3x - 5, & x < 1 \\
-2x + 3, & x \geq 1
\end{cases}
\]

1. Find \( f(-4) \):
   Since \(-4 < 1\), the function \(3x - 5\) will be used.
   \[
   f(x) = 3x - 5 \\
f(-4) = 3(-4) - 5 \\
f(-4) = -12 - 5 \\
f(-4) = -17 \text{ or } (-4, -17)
   \]

2. Find \( f(3) \):
   Since \(3 > 1\), the function \(-2x + 3\) will be used.
   \[
   f(x) = -2x + 3 \\
f(3) = -2(3) + 3 \\
f(3) = -6 + 3 \\
f(3) = -3 \text{ or } (3, -3)
   \]

3. Find \( f(1) \):
   Since \(1 \geq 1\), the function \(-2x + 3\) will be used.
   \[
   f(x) = -2x + 3 \\
f(1) = -2(1) + 3 \\
f(1) = -2 + 3 \\
f(1) = 1 \text{ or } (1, 1)
   \]
In-Class Practice:

4. \( f(x) = \begin{cases} 
|x + 3| - 8, & x < -2 \\
-2x - 3, & -2 \leq x \leq 1 \\
(x - 4)^2, & x \geq 1 
\end{cases} \)

a. \( f(4) \)

b. \( f(-1) \)

c. \( f(-5) \)

5. \( f(x) = \begin{cases} 
|x| + 5, & x < -4 \\
(x + 1)^2 - 2, & -4 \leq x \leq 4 \\
2x - 1, & x > 4 
\end{cases} \)

a. \( f(4) \)

b. \( f(-2) \)

c. \( f(-7) \)

d. \( f(7.5) \)
Piecewise Functions – Substituting
HW
Quadratic Unit
Algebra 1

Draw and label a number line for each function.

1. \( f(x) = \begin{cases} 
  x - 7, & x < -8 \\
  (x + 8)^2 - 3, & x \geq -8 
\end{cases} \)
   a. \( f(-2) \)
   c. \( f(10) \)
   b. \( f(-12) \)
   d. \( f(20) \)

2. \( f(x) = \begin{cases} 
  2(x + 5)^2 + 1, & x < -4 \\
  12, & -4 \leq x < 6 \\
  -\frac{1}{2}x + 10, & x \geq 6 
\end{cases} \)
   a. \( f(-5) \)
   c. \( f(-10) \)
   b. \( f(5) \)
   d. \( f(10) \)
3. \( f(x) = \begin{cases} 
-4, & x < 0 \\
2x - 1, & 0 \leq x < 25 \\
-(x - 25)^2 + 50, & x \geq 25 
\end{cases} \)

a. \( f(21) \)  
d. \( f(-18) \)

b. \( f(26) \)  
e. \( f(40) \)

c. \( f(0) \)  
f. \( f(14) \)
Example: Graph the piecewise function.

\[ f(x) = \begin{cases} 
 3x - 5, & \quad x < 1 \\
 -2x + 3, & \quad x \geq 1 
\end{cases} \]

Start by graphing and labeling each line on the graph.

Draw vertical lines at all of the restrictions for \( x \).

Erase the sections of the graph where the functions do not exist. Use open and closed circles on the endpoints based on the inequality symbol.
In-Class Practice:

1. \( f(x) = \begin{cases} 
|x + 4|, & x < -2 \\
x^2 - 1, & -2 \leq x \leq 2 \\
2x - 3, & x > 2 
\end{cases} \)

2. \( f(x) = \begin{cases} 
-7, & x < -1 \\
x^2 - 4, & -1 \leq x < 3 \\
|x - 4| - 10, & x \geq 3 
\end{cases} \)
Piecewise Functions – Graphing

1. \( f(x) = \begin{cases} 
-2x + 3, & x \leq 4 \\
(x - 2)^2 - 3, & x > 4 
\end{cases} \)

2. \( f(x) = \begin{cases} 
\frac{3}{5}x - 2, & x < 0 \\
(x + 1)^2 - 1, & x \geq 0 
\end{cases} \)

3. \( f(x) = \begin{cases} 
\lceil x + 5 \rceil, & x \leq -2 \\
x^2, & -2 < x \leq 3 \\
5, & 3 < x 
\end{cases} \)

4. \( f(x) = \begin{cases} 
-2, & x < -3 \\
(x + 3)^2 + 2, & -3 \leq x \leq -1 \\
-4, & -1 < x 
\end{cases} \)
Piecewise Functions – Writing Equations

Example: Write the equation of the function.

\[ f(x) = \begin{cases} 
\frac{-4}{3}x - 3, & x \leq -6 \\
\frac{1}{6}x + 5, & -6 < x \leq 6 \\
\frac{1}{3}x, & 6 < x 
\end{cases} \]

Extend the lines to determine the slopes and y-intercepts. Use vertical lines to find the domain for each piece of the function.

In-Class Practice:
1. Construct the equation of the function.
2. Construct the equation of the function.
Write the equation for each function:

1. 

2. 

3. 

RHS Mathematics Department
Expand each of the following.

1. $(x + 2)(x + 5)$  
2. $(x + 7)(x - 1)$  

3. $(x + 4)(x + 10)$  
4. $(x + 5)(x - 5)$  

5. $(x + y)(x + 3)$  
6. $(4x - 3z)(2y - 5w)$  

7. $(2x + 5)(x + 7)$  
8. $(x + 6)^2$  

9. $(x - 3)(4x + 9)$  
10. $(x - 12)^2$
11. $(4x - 11)(4x + 11)$
12. $(x + 9)(x - 8)$

13. $(2x - 7)(2x - 7)$
14. $(x + 4)^2$

15. $(x - 1)(x - 8)$
16. $(x + 15)(x - 15)$

17. $(6x - 7)(x + 2)$
18. $(9x - w)(7z - 2y)$

19. $(2x + 3y)(x + 5)$
20. $(7x + 2)(3x - 1)$
21. \((x + 1)(x + 6)\)  
22. \((x + 6)(x - 2)\)

23. \((x + 3)(x + 11)\)  
24. \((x + 8)(x - 8)\)

25. \((x + y)(2x + 5)\)  
26. \((3x - 2z)(3y - 7w)\)

27. \((2x + 3)(x + 8)\)  
28. \((x + 9)^2\)

29. \((x - 2)(5x + 12)\)  
30. \((x - 14)^2\)
31. $(3x - 10)(5x + 12)$  
32. $(x + 8)(x - 7)$  

33. $(6x - 5)(6x - 5)$  
34. $(x + 13)^2$  

35. $(x - 2)(x - 13)$  
36. $(x + 9)(x - 9)$  

37. $(5x - 6)(x + 3)$  
38. $(8x - 3w)(6z + 7y)$  

39. $(4x + 7y)(x + 2)$  
40. $(6x + 1)(4x - 3)$
Find the greatest common factor of each set.

1. 84, 78  
2. 72, 87  
3. 22, 88  
4. 68x, 48xy  
5. 99, 39  
6. 12x^2y, 72xy^3
<p>| | | |</p>
<table>
<thead>
<tr>
<th></th>
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</thead>
<tbody>
<tr>
<td>7.</td>
<td>90, 12</td>
<td>8.</td>
</tr>
<tr>
<td>10.</td>
<td>90x, 70y</td>
<td>11.</td>
</tr>
<tr>
<td>13.</td>
<td>7, 56xy, 42y</td>
<td>14.</td>
</tr>
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<td>---</td>
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<td>---</td>
</tr>
<tr>
<td>16.</td>
<td>$67xy^3, 44y^2$</td>
<td>17.</td>
</tr>
<tr>
<td>19.</td>
<td>$42, 35$</td>
<td>20.</td>
</tr>
</tbody>
</table>
22. $80xy^2, 72x$
23. $32xy, 88x, 24yz$
24. 96, 12

25. $36x^4y^3, 63xy^5$
26. 68, 76, 96
27. $96xy, 24xy^2, 72y^3$
Factoring by Grouping
Supplemental Problems
Quadratic Unit
Algebra 1

1. \(18x^2 - 4x - 63xy + 14y\)

2. \(15x^2 - 25x + 9xy - 15y\)

3. \(35x^2 - 14x + 5xy - 2y\)
4. \( 4x^2 + 8x - xy - 2y \)

5. \( 2x^2 + 6x + 9xy + 27y \)

6. \( 7x^2 - 6x - 56xy + 48y \)

7. \( x^2 + x - 4xy - 4y \)
Factor by Reverse Distributing.

1. $6x^2 - 10x$

2. $-18x^3 + 3x^2$

3. $20x^2y + 15xy^2$

4. $16x^2 - 32x$

5. $-8x^2y - 28x^2$

6. $18x^3y^3 + 3x^2y^5$
Factor by Grouping.

7. $7x^2 + 6x + 70xy + 60y$

8. $25x^2 - 50x + 30xy - 60y$

9. $9x^2 + 36x + 4xy + 16y$

10. $63x^2 - 14x - 18xy + 4y$

11. $9x^2 + 24x - 24xy - 64y$

12. $30x^2 - 24x - 5xy + 4y$
Factor Trinomials.

13. $x^2 + 12x + 32$
14. $2x^2 - 6x - 20$

15. $x^2 + 2x - 15$
16. $x^2 - 11x + 28$

17. $3x^2 - 7x + 4$
18. $2x^2 + 15x + 18$
Factor by Reverse Distributing.

1. $4x^3 - 12x^2$

2. $12x^2 - 36x$

3. $24x^4 + 3x^2$

4. $-3x^2y^2 - 21x^2y$

5. $72y + 18xy^2$

6. $-88xy^3 + 11x^2y^5$
Factor by Grouping.
7. \(2x^2y + 6xy - x - 3\)
8. \(x^3 + xy^2 - x^2y - y^3\)
9. \(4x^2y - 8xy - 3x + 6\)
10. \(xz + xw + yz + yw\)
11. \(3y^2 - 2y + 12ky - 8k\)
12. \(3j - 5j^2 - 6k + 10jk\)

Factor Trinomials.
13. \(x^2 + 5x - 24\)
14. \(3x^2 + 2x - 16\)
15. $x^2 - 4x - 12$  
16. $x^2 - 18x + 80$

17. $2x^2 + 4x - 30$  
18. $x^2 + 24x + 144$

Factor Difference of Squares.

19. $4x^2 - 25$  
20. $9x^2 - 144$

21. $25x^2 - 121$  
22. $16x^2 - 9$

23. $x^2 - 64$  
24. $4x^2 - 49$
Linear, Quadratic Or Neither?  
Supplemental Problems  
Quadratic Unit  
Algebra 1  

Determine if each function is Linear, Quadratic or Neither. If it is Linear or Quadratic, give the equation.

1. | Δx | x | y | 1st difference | 2nd difference |
<table>
<thead>
<tr>
<th></th>
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<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>-4</td>
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<td>1</td>
<td>37</td>
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</tbody>
</table>

2. | x | y | 1st difference |
<table>
<thead>
<tr>
<th></th>
<th></th>
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</thead>
<tbody>
<tr>
<td>-2</td>
<td>-13</td>
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</tr>
<tr>
<td>-1</td>
<td>-9</td>
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<td>1</td>
<td>-1</td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>-1</td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>7</td>
<td></td>
</tr>
</tbody>
</table>
3. \[ \begin{array}{cccccc}
\begin{array}{c}
\text{x} \\
\text{y}
\end{array}
\begin{array}{cccccc}
0 & 1 & 2 & 3 & 4 & 5 \\
\frac{1}{2} & 1 & 2 & 4 & 8 & 16
\end{array}
\end{array} \]

4. \[ \begin{array}{cccccc}
\begin{array}{c}
\text{x} \\
\Delta x
\end{array}
\begin{array}{cccccc}
-3 & -2 & -1 & 0 & 1 & 2 \\
-12 & -21 & -22 & -15 & 0 & 23
\end{array}
\end{array} \]

5. \[ \begin{array}{cccccc}
\begin{array}{c}
\text{x} \\
\text{y}
\end{array}
\begin{array}{cccccc}
0 & 1 & 2 & 3 & 4 & 5 \\
-3 & 15 & 35 & 57 & 81 & 107
\end{array}
\end{array} \]
6. 

<table>
<thead>
<tr>
<th>x</th>
<th>y</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>1</td>
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<tr>
<td>1</td>
<td>3</td>
</tr>
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<td>2</td>
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<tr>
<td>3</td>
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<tr>
<td>4</td>
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7. 

<table>
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<tr>
<th>x (Δx)</th>
<th>y</th>
<th>1\textsuperscript{st} difference</th>
<th>2\textsuperscript{nd} difference</th>
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8. 

<table>
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<th>x</th>
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<td>39</td>
<td>32</td>
<td>25</td>
<td>18</td>
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9. 

<table>
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<th>-1</th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
</tr>
</thead>
<tbody>
<tr>
<td>y</td>
<td>32</td>
<td>19</td>
<td>12</td>
<td>11</td>
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</table>
For questions #1-3, determine if each function is Linear, Quadratic or Neither. If it is Linear or Quadratic, give the equation.

### Question 1

<table>
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<tr>
<th>$\Delta x$</th>
<th>$x$</th>
<th>$y$</th>
<th>$1^{st}$ difference</th>
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Equation: ___________________________

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Equation: ___________________________
3. 

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<th>$y$</th>
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<tr>
<td>4</td>
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</table>

| Linear | Quadratic | Neither |

Equation: _____________________________

For questions #4-5, simplify each of the following by using the double distributive method.

4. $(3x + 6)(x - 2)$

5. $(2x - 9y)(x + 2)$

For questions #6-17 factor each of the following using reverse distributive, trinomials, grouping, or difference of squares.

6. $9x^2 - 225$

7. $10x + 15 - 4xy - 6y$

8. $16x^2 - 36$

9. $2x^2 + 8$
10. \(3x^2 + 19x - 14\)  
11. \(64x^2 - 81\)  

12. \(x^2 + 7x - 18\)  
13. \(25x^2 - 50\)  

14. \(2x^2 + x - 21\)  
15. \(4x^2 - 4\)  

16. \(2x^2 + 2x - 12\)  
17. \(24x^5 + 6x^3\)
18. Complete the following:

a. Graph \( y = x + 5 \) on the grid and in \( Y_1 \).

b. Graph \( y = x + 3 \) on the grid and in \( Y_2 \).

c. Complete the \( y \)-values in the table below algebraically or using TABLE. Then use the \( y \)-values you found to complete the bottom row of the table.

<table>
<thead>
<tr>
<th>( x )-values</th>
<th>-6</th>
<th>-5</th>
<th>-4</th>
<th>-3</th>
<th>-2</th>
<th>-1</th>
<th>0</th>
</tr>
</thead>
<tbody>
<tr>
<td>( y )-values for ( x + 5 )</td>
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<td></td>
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<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>( y )-values for ( x + 3 )</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Multiply ( y )-values for ( x + 5 ) and ( x + 3 ) together</td>
<td></td>
<td></td>
<td></td>
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<td></td>
<td></td>
</tr>
</tbody>
</table>

d. Make a list of coordinates where \( x = \) the \( x \)-value from the table and \( y = \) the multiplied value you found. Plot and connect these points on the grid above.

e. Write the vertex form of the quadratic function.

f. Change from vertex form to standard form.

g. Double distribute the two linear functions. What do you notice about this equation and the standard form equation?

h. Solve \( x + 5 = 0 \) and \( x + 3 = 0 \).

i. Where does the quadratic function have \( x \)-intercepts?
For questions #1-3, determine if each function is Linear, Quadratic or Neither. If it is Linear or Quadratic, give the equation.

1. \[ \begin{array}{c|c|c|c}
\Delta x & x & y & 1^{st} \text{ difference} \\
\hline
0 & 8 & & \\
1 & 9 & & \\
2 & 16 & & \\
3 & 29 & & \\
4 & 48 & & \\
5 & 73 & & \\
\end{array} \]

   \[ \begin{array}{c|c|c|c}
& & 2^{nd} \text{ difference} & \\
\hline
& & & \\
& & & \\
& & & \\
& & & \\
& & & \\
& & & \\
\end{array} \]

   Equation: ___________________________

2. \[ \begin{array}{c|c|c|c}
\Delta x & x & y & 1^{st} \text{ difference} \\
\hline
-2 & -8 & & \\
-1 & -5 & & \\
0 & -2 & & \\
1 & 1 & & \\
2 & 4 & & \\
3 & 7 & & \\
\end{array} \]

   \[ \begin{array}{c|c|c|c}
& & 2^{nd} \text{ difference} & \\
\hline
& & & \\
& & & \\
& & & \\
& & & \\
& & & \\
& & & \\
\end{array} \]

   Equation: ___________________________
3. | Δx | x | y | 1<sup>st</sup> difference | 2<sup>nd</sup> difference | Linear | Quadratic
---|---|---|---|---|---|---
-1 | 3 | 0 | 2 | 1 | Neither
0 | 2 |
1 | 1 |
2 | -6 |
3 | -25 |
4 | -62 |

Equation: _____________________________

For questions #4-5, simplify each of the following by using the double distributive method.

4. (3x + 6)(x – 2)  
5. (2x – 9y)(x + 2)

For questions #6-17 factor each of the following using reverse distributive, trinomials, grouping, or difference of squares.

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12. \(x^2 + 7x - 18\)  \hspace{1cm} 13. \(25x^2 - 50\)

14. \(2x^2 + x - 21\)  \hspace{1cm} 15. \(4x^2 - 4\)

16. \(2x^2 + 2x - 12\)  \hspace{1cm} 17. \(24x^5 + 6x^3\)
18. Complete the following:

a. Graph \( y = x + 5 \) on the grid and in \( Y_1 \).

b. Graph \( y = x + 3 \) on the grid and in \( Y_2 \).

c. Complete the \( y \)-values in the table below algebraically or using TABLE. Then use the \( y \)-values you found to complete the bottom row of the table.

<table>
<thead>
<tr>
<th>( x )-values</th>
<th>-6</th>
<th>-5</th>
<th>-4</th>
<th>-3</th>
<th>-2</th>
<th>-1</th>
<th>0</th>
</tr>
</thead>
<tbody>
<tr>
<td>( y )-values for ( (x + 5) )</td>
<td></td>
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<td></td>
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<tr>
<td>( y )-values for ( (x + 3) )</td>
<td></td>
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<td></td>
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<tr>
<td>Multiply ( y )-values for ( (x + 5) ) and ( (x + 3) ) together</td>
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<td></td>
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</tbody>
</table>

d. Make a list of coordinates where \( x = \) the \( x \)-value from the table and \( y = \) the multiplied value you found. Plot and connect these points on the grid above.

e. Write the vertex form of the quadratic function.

f. Change from vertex form to standard form.

g. Double distribute the two linear functions. What do you notice about this equation and the standard form equation?

h. Solve \( x + 5 = 0 \) and \( x + 3 = 0 \).

i. Where does the quadratic function have \( x \)-intercepts?
Find the product of the multiplication of the binomials by double distributing.
1. $(3w - 7)(4w + 9)$  
2. $(4x - 8)(2x + 3)$  
3. $(x^2 - 4)(x^2 + 4)$

4. $(4xy - 2)(2xy + 6)$  
5. $\left(\frac{3}{2}n - 4\right)\left(\frac{5}{2}n + 2\right)$  
6. $(12m - 3)^2$

Determine if the function is linear, quadratic or neither. If it is linear or quadratic, give the equation.

7. Type:____________________

<table>
<thead>
<tr>
<th>$\Delta x$</th>
<th>$x$</th>
<th>$y$</th>
<th>$1^{st}$ Difference</th>
<th>$2^{nd}$ Difference</th>
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<tr>
<td>5</td>
<td>5</td>
<td>140</td>
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<td></td>
</tr>
</tbody>
</table>

Equation:____________________________
List what type of factoring you would use for each, then factor each completely.

8. $n^2 - 25$ Type:_______________ 9. $15p^3q^2 - 45pr$ Type:_______________

10. $9x^2 - 27$ Type:_______________ 11. $x^2 - 4x - 12$ Type:_______________

12. $6x^2 - 4xy + 12xy - 8y^2$ Type:_______________ 13. $8x^4 - 144$ Type:_______________

Simplify each radical expression.

14. $\sqrt{450}$ 15. $5 \cdot \sqrt{54}$ 16. $\frac{\sqrt{33} \cdot \sqrt{121}}{11}$

17. $5\sqrt{18} \cdot 2\sqrt{8}$ 18. $\frac{\sqrt{63}}{\sqrt{45}}$ 19. $\frac{2\sqrt{18} - 4\sqrt{6}}{3\sqrt{5} - 7\sqrt{15}}$
Solve each quadratic equation by factoring.
20. \( f(x) = x^2 + 3x - 10 \) \hspace{1cm} 21. \( f(x) = 2x^2 + 11x + 15 \)

Solve by graphing.
22. \( f(x) = -2x^2 + 4x \) \hspace{1cm} 23. \( f(x) = x^2 - 4x + 3 \) \hspace{1cm} 24. \( f(x) = -x^2 - 6x - 5 \)

Given the solutions, find the quadratic equation for the function in standard form (no fractions).
25. \((-5,0), (3,0)\) and the quadratic function opens down.

Equation: ______________________

26. \((2,0)\) and the quadratic function opens up.

Equation: ______________________

27. \( \left( \frac{2}{3}, 0 \right) \) and the quadratic function opens up.

Equation: ______________________
28. \( y = -(x + 3)^2 + 1 \)

a. Graph the function

b. What are the zeros?

c. What two lines make up this quadratic?
   Linear 1: ____________  Linear 2: ____________

d. Write the function in factored form.

e. Write the function in standard form.

f. What is the y-intercept?

g. Draw and give the equation of the line of symmetry.

h. Identify the type of critical point and give the location.

i. Describe the behavior of the function.

j. Graph the inverse of \( y = -(x + 3)^2 + 1 \) to the right.

k. List the domain and range of the original and inverse functions.

   \[
   \begin{align*}
   \text{ORIGINAL} & \rightarrow \text{Domain:} & \text{Range:} \\
   \text{INVERSE} & \rightarrow \text{Domain:} & \text{Range:}
   \end{align*}
   \]
29. The Fray just released a new album and is headed to the Palace of Auburn Hills to kick off their new tour. The concert is the talk of the Metro-Detroit area and the owner of the Palace knows that he can increase revenue by raising the price of tickets. He focuses on the lower bowl seating sections.

The owner of the Palace decides to do a revenue-cost-profit analysis. To compute the money the concert will take in (revenue or \( R \)), they must multiply the number of people (attendance or \( A \)), and the amount they charge per person, (ticket price or \( T \)); so, \( R = A \cdot T \)

The owner knows that less people will come the more he raises the price per ticket. They conduct a poll of many of the Fray’s fans and estimate that they will lose 30 people for every \$0.35 they raise the price. They have heard from rumors that the show will sell out the lower bowl, attracting 8,000 people if they charge \$50 for a ticket. (This is the normal cost per ticket)

- Let \( x \) represent the number of times the price is changed
- Let \( A \) represent the attendance
- Let \( T \) represent the ticket price
- Let \( R \) represent the revenue for a given value of \( x \)

Fill in the table showing attendance, ticket price, and revenue for \( x \) values from 0 to 4.

<table>
<thead>
<tr>
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<th>2</th>
<th>3</th>
<th>4</th>
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<td></td>
</tr>
<tr>
<td>Ticket Price ( (T) )</td>
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<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Revenue ( (R) )</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

a. Write a rule for the attendance versus \# of price changes \( (A \text{ in terms of } x) \).

b. Write a rule for the ticket price versus \# of price changes \( (T \text{ in terms of } x) \).

c. Write a rule for the revenue versus \# of price changes \( (R \text{ in terms of } x) \).

d. What is the best possible revenue? 

e. What should the ticket price be?

f. How many people will come?
Quadratic Review #3
Supplemental Problems
Quadratic Unit
Algebra 1

Simplify.
1. \(\sqrt{99}\)  
2. \(\sqrt{704}\)

3. \(\sqrt{1250}\)  
4. \(\sqrt{5 \cdot 60}\)

5. \(11\sqrt{14} \cdot 2\sqrt{7}\)  
6. \(\frac{\sqrt{12}}{\sqrt{36}}\)

7. \(\frac{2\sqrt{18}}{\sqrt{20}}\)  
8. \(\frac{\sqrt{5} \cdot \sqrt{24}}{\sqrt{32} \cdot \sqrt{2}}\)
Solve by factoring.
9. \( f(x) = x^2 + 17x + 52 \)  
10. \( f(x) = 15x^2 - x - 2 \)

11. \( f(x) = x^2 + 5x - 84 \)  
12. \( f(x) = 6x^2 - 5x - 4 \)

Solve by graphing, round to the nearest hundredth.
13. \( f(x) = -4x^2 + 9x + 24 \)  
14. \( f(x) = x^2 - 7x - 10 \)

Given the solutions, find a quadratic equation in standard form (no fractions).
15. (-7, 0), (3, 0) and the quadratic function opens down.

16. (6, 0) and the quadratic function opens up.

17. (4, 0), \((-2/5, 0)\) and the quadratic function opens down.
Given a table of values, find a quadratic equation that fits the data points in **factored form** then in **standard form**. Assume the quadratic opens up.

18.  

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19.  

<table>
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<td>-1</td>
<td>2</td>
</tr>
<tr>
<td>0</td>
<td>6</td>
</tr>
</tbody>
</table>

Write in standard form.

20.  \( y = (x + 5)^2 - 7 \)

21.  \( f(x) = -(x + 7)^2 + 13 \)

22.  \( f(x) = \frac{1}{2} (x + 8)^2 - 5 \)

23.  \( y = (x - 6)^2 + 3 \)
24. There is a buzz in the air as the Tigers get ready to begin the new season. Mike Illitch wants to maximize his profits this season so that he can keep bringing in players to help the team win the World Series. Mr. Illitch decides to focus on the bleacher seats to maximize the ticket sales. Last season the bleacher seats cost $5 per game. Mike decides to survey the season ticket holders and finds that they will lose 25 people for every $0.50 they raise the price. They know from last year’s sales they will sell 2,000 tickets when charging $5 per game.

- Let \( x \) represent the number of times the price is changed
- Let \( A \) represent the attendance
- Let \( T \) represent the ticket price
- Let \( R \) represent the revenue for a given value of \( x \)

a. Complete the table below.

<table>
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<th>-3</th>
<th>-2</th>
<th>-1</th>
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<th>3</th>
</tr>
</thead>
<tbody>
<tr>
<td>Attendance (( A ))</td>
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<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Ticket Price (( T ))</td>
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<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Revenue (( R ))</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

b. Write a rule for the attendance versus number of price changes.

c. Write a rule for the ticket price versus number of price changes.

d. Write a rule for the revenue versus number of price changes.

e. What is the best possible revenue?

f. How many price changes does the maximum revenue happen at?

g. What should be the ticket price? How many people will come?
For questions #1-8, simplify each radical.

1. $\sqrt{242}$
2. $3\sqrt{20} \cdot 6\sqrt{15}$

3. $\frac{\sqrt{18}}{\sqrt{2}}$
4. $5\sqrt{250}$

5. $2\sqrt{10} \cdot \sqrt{75}$
6. $\sqrt{507}$

7. $5\sqrt{27} \cdot 2\sqrt{12}$
8. $\frac{\sqrt{147}}{\sqrt{7}}$
For questions #9-13, factor each polynomial completely.

9. \( 9xy + 8x + 63y + 56 \)

10. \( 50x^2 - 18 \)

11. \( 7x^2 - 3x - 4 \)

12. \( 24x^2y - 12x^3y^3 + 6x^2y^5 \)

13. \( 49 - 121x^2 \)
Write the following factored form quadratics in standard form.

1. \( y = (x + 4)(x - 1) \)  
2. \( y = (2x - 9)(x + 1) \)  
3. \( f(x) = (x - 12)(x - 7) \)  
4. \( f(x) = (4x + 3y)(x - 5) \)

Determine if each function is Linear, Quadratic or Neither. If it is Linear or Quadratic, give the equation.

5. | \( \Delta x \) | \( x \) | \( y \) | 1st difference | 2nd difference | Linear | Quadratic | Neither |
<table>
<thead>
<tr>
<th></th>
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<td></td>
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<tr>
<td>4</td>
<td>27</td>
<td></td>
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<tr>
<td>5</td>
<td>42</td>
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</tbody>
</table>

Equation: _______________________________
6. 

<table>
<thead>
<tr>
<th>x</th>
<th>y</th>
<th>1st difference</th>
<th>2nd difference</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>-2</td>
<td></td>
<td></td>
</tr>
<tr>
<td>1</td>
<td>7</td>
<td></td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>24</td>
<td></td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>49</td>
<td></td>
<td></td>
</tr>
<tr>
<td>4</td>
<td>82</td>
<td></td>
<td></td>
</tr>
<tr>
<td>5</td>
<td>123</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Δx</th>
<th>x</th>
<th>y</th>
<th>Linear</th>
<th>Quadratic</th>
</tr>
</thead>
</table>

Neither

Equation: ________________________________

Factor each of the following using reverse distribution, trinomials, grouping, or difference of squares.

7. $x^2 - 121$  
8. $2x^3 + 12x^2y + 8xy^2$

9. $x^2 + 4x - 21$  
10. $2x^2 + 9x + 4$

11. $4x^2 + 5x - 28xy - 35y$  
12. $x^2 - 12x + 35$
Simplify the following. Leave in radical form.

13. $\sqrt{20}$

14. $\sqrt{47}$

15. $3\sqrt{54}$

16. $7\sqrt{21} \cdot 11\sqrt{51}$

17. $2\sqrt{242} \cdot 10\sqrt{42}$

18. $\sqrt{72}$

19. $\frac{7\sqrt{50}}{\sqrt{49}}$

20. $\frac{\sqrt{44}}{3}$
21. Complete the following:

   a. Graph \( y = x + 4 \) on the grid and in \( Y_1 \).

   b. Graph \( y = x + 2 \) on the grid and in \( Y_2 \).

   c. Complete the \( y \)-values in the table below algebraically or using TABLE. Then use the \( y \)-values you found to complete the bottom row of the table.

<table>
<thead>
<tr>
<th>( x )-values</th>
<th>-6</th>
<th>-5</th>
<th>-4</th>
<th>-3</th>
<th>-2</th>
<th>-1</th>
<th>0</th>
</tr>
</thead>
<tbody>
<tr>
<td>( y )-values for ( x + 4 )</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>( y )-values for ( x + 2 )</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Multiply ( y )-values for ( x + 4 ) and ( x + 2 ) together</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

   d. Make a list of coordinates where \( x \) = the \( x \)-value and \( y \) = the multiplied value.

   e. Plot and connect these points on the grid above.

   f. Write the vertex form of the quadratic function.

   g. Double distribute the two linear functions.

   h. Change from standard form to vertex form.

   i. Solve \( x + 4 = 0 \) and \( x + 2 = 0 \).

   j. Where does the quadratic equation have \( x \)-intercepts?
22. Romeo High School wants to put on a fall choir concert. They would like to at least cover the costs of putting on the show and ideally; they would like to make a profit so that they can put on a more elaborate show in the spring.

After looking over the receipts from the last few concerts, Mr. Hinkle noticed that the more they charge, the fewer people come. They conduct a poll of several classes and estimate that they will lose 10 people for every $0.50 they raise the price. They know from past experience that the concert will attract 450 people if they charge $7.50 for a ticket.

a. Let \( x \) represent the number of times the price is changed
   - Let \( A \) represent the attendance
   - Let \( T \) represent the ticket price
   - Let \( R \) represent the revenue for a given value of \( x \)

b. Fill in the table showing attendance, ticket price, and revenue for \( x \) values from -4 to 4.

<table>
<thead>
<tr>
<th>( x )</th>
<th>-4</th>
<th>-3</th>
<th>-2</th>
<th>-1</th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
</tr>
</thead>
<tbody>
<tr>
<td>Attendance ((A))</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Ticket Price ((T))</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Revenue ((R))</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

c. Write a rule for the attendance versus number of price changes \((A \text{ in terms of } x)\).

d. Write a rule for the ticket price versus number of price changes \((T \text{ in terms of } x)\).

e. Write a rule for the revenue versus number of price changes \((R \text{ in terms of } x)\).
   Remember that \( R = A \cdot T \).
f. Use your graphing calculator to find the best price to charge. Remember that \( x \) represents the number of price changes that occur, not dollars or people.

1. What is the best possible revenue? How many price changes will take place?

2. What should the ticket price be?

3. How many people will come?

Find the factored form and standard form of quadratic function.

23. \((4,0), (-7,0), \text{ opens up}\) 

24. \((-3,0), (-5,0), \text{ opens down}\)

<table>
<thead>
<tr>
<th>(x)</th>
<th>(y)</th>
</tr>
</thead>
<tbody>
<tr>
<td>-5</td>
<td>6</td>
</tr>
<tr>
<td>-4</td>
<td>0</td>
</tr>
<tr>
<td>-3</td>
<td>-4</td>
</tr>
<tr>
<td>-2</td>
<td>-6</td>
</tr>
<tr>
<td>-1</td>
<td>-4</td>
</tr>
<tr>
<td>0</td>
<td>0</td>
</tr>
</tbody>
</table>

(opens up)

<table>
<thead>
<tr>
<th>(x)</th>
<th>(y)</th>
</tr>
</thead>
<tbody>
<tr>
<td>-2</td>
<td>0</td>
</tr>
<tr>
<td>-1</td>
<td>4</td>
</tr>
<tr>
<td>0</td>
<td>6</td>
</tr>
<tr>
<td>1</td>
<td>6</td>
</tr>
<tr>
<td>2</td>
<td>4</td>
</tr>
<tr>
<td>3</td>
<td>0</td>
</tr>
</tbody>
</table>

(opens down)
What value for “c” makes the quadratic a perfect square trinomial?

27. \( y = x^2 + 32x + c \)  
28. \( y = x^2 - 18x + c \)

29. \( y = x^2 + 14x + c \)  
30. \( y = x^2 - 24x + c \)

Write each quadratic in vertex form, then give the vertex.

31. \( y = x^2 + 16x + 13 \)  
32. \( y = x^2 + 26x - 18 \)

Vertex: _______________  
Vertex: _______________

33. \( y = x^2 - 34x - 23 \)  
34. \( y = x^2 - 22x + 17 \)

Vertex: _______________  
Vertex: _______________
For questions #1-8, solve for x and simplify radicals where necessary.

1. \((x - 5)^2 = 81\) 
2. \(2x^2 - 4 = 28\)

3. \((x + 1)^2 + 3 = 52\) 
4. \(2(x + 3)^2 - 4 = 68\)

5. \(5(x - 1)^2 = 500\) 
6. \(4x^2 - 2 = 14\)

7. \(2(x + 7)^2 - 8 = 234\) 
8. \(x^2 = 972\)
For questions #9-11, solve each quadratic by graphing. Draw a sketch of the graph.

9. \( y = 3x^2 - 2x - 5 \)  
10. \( f(x) = -5x^2 + 4x + 7 \)  
11. \( y = -x^2 - 12.5x - 39.0625 \)

Solution(s):

Solution(s):

Solution(s):

For questions #12-17, solve each quadratic by factoring. Be sure to confirm your solutions graphically on your calculator.

12. \( y = 2x^2 - 8 \)  
13. \( f(x) = x^2 - 5x - 14 \)

14. \( f(x) = 3x^2 + x - 2 \)  
15. \( y = 49x^2 - 9 \)

16. \( y = 4x^2 + 7x - 2 \)  
17. \( f(x) = x^2 - 81 \)
For questions #18-21, given the following information, find the quadratic equation in both factored form and standard form.

18. Solutions: (2,0), (−4,0) and the quadratic function opens up.

19. The quadratic opens up and contains the following points:

<table>
<thead>
<tr>
<th>x</th>
<th>-2</th>
<th>-1</th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
</tr>
</thead>
<tbody>
<tr>
<td>y</td>
<td>5</td>
<td>0</td>
<td>-3</td>
<td>-4</td>
<td>-3</td>
<td>0</td>
<td>5</td>
</tr>
</tbody>
</table>

20. Solutions: (−5,0), (1,0) and the quadratic function opens down.

21. The quadratic opens down and contains the following points:

<table>
<thead>
<tr>
<th>x</th>
<th>-10</th>
<th>-9</th>
<th>-8</th>
<th>-7</th>
<th>-6</th>
<th>-5</th>
<th>-4</th>
</tr>
</thead>
<tbody>
<tr>
<td>y</td>
<td>-15</td>
<td>-8</td>
<td>-3</td>
<td>0</td>
<td>1</td>
<td>0</td>
<td>-3</td>
</tr>
</tbody>
</table>
For questions #22-24,
a. Write in vertex form  
b. Give the vertex  
c. Find the equation for the AOS 
d. Find the x-intercepts  
e. Write in factored form  
f. Give the y-intercept

22. \( y = x^2 - 4x - 5 \)

a. ______________________
b. ______________________
c. ______________________
d. ______________________
e. ______________________
f. ______________________

23. \( y = x^2 + 8x \)

a. ______________________
b. ______________________
c. ______________________
d. ______________________
e. ______________________
f. ______________________

24. \( y = x^2 - 20x + 96 \)

a. ______________________
b. ______________________
c. ______________________
d. ______________________
e. ______________________
f. ______________________
Simplify each radical.

1. \(-\sqrt{-150}\)
2. \(-3\sqrt{-7} \cdot 6\sqrt{-7}\)
3. \(\sqrt{-125}\)
4. \(\sqrt{5} \cdot \sqrt{-10}\)
5. \(-\sqrt{192}\)
6. \(\sqrt{-5} \cdot -4\sqrt{20}\)
7. \(\sqrt{-15} \cdot \sqrt{10}\)
8. \(\sqrt{-18} \cdot 4\sqrt{-3}\)
Solve for x, simplifying radicals when necessary.

9. \( x^2 + 7 = 88 \)

10. \( 5x^2 - 7 = 488 \)

11. \( -7x^2 = -448 \)

12. \( 4x^2 + 1 = 325 \)

Solve each equation below. Simplify radicals when necessary.

13. \( 2n^2 - 20 = -84 \)

14. \( 9y^2 = -81 \)

15. \( -4p^2 + 10 = 266 \)
Complete the table.

<table>
<thead>
<tr>
<th>Vertex Form</th>
<th>1. f(x) = (x – 2)^2 – 9</th>
<th>2.</th>
<th>3.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Standard Form</td>
<td>y = x^2 – 14x + 48</td>
<td>2.</td>
<td>3.</td>
</tr>
<tr>
<td>Factored Form</td>
<td>f(x) = (x – 6)(x – 10)</td>
<td>2.</td>
<td>3.</td>
</tr>
<tr>
<td>LOS</td>
<td>2.</td>
<td>3.</td>
<td></td>
</tr>
<tr>
<td>Vertex</td>
<td>2.</td>
<td>3.</td>
<td></td>
</tr>
<tr>
<td>Solutions</td>
<td>2.</td>
<td>3.</td>
<td></td>
</tr>
<tr>
<td>y-intercept</td>
<td>2.</td>
<td>3.</td>
<td></td>
</tr>
<tr>
<td></td>
<td>4.</td>
<td>5.</td>
<td>6.</td>
</tr>
<tr>
<td>---------------</td>
<td>-----------------------------------------</td>
<td>-----------------------------------------</td>
<td>-----------------------------------------</td>
</tr>
<tr>
<td><strong>Vertex Form</strong></td>
<td><strong>f(x) = (x + 6)^2 - 4</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>Standard Form</strong></td>
<td></td>
<td><strong>y = x^2 + 22x + 40</strong></td>
<td></td>
</tr>
<tr>
<td><strong>Factored Form</strong></td>
<td></td>
<td></td>
<td><strong>f(x) = (x - 2)(x - 8)</strong></td>
</tr>
<tr>
<td><strong>LOS</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>Vertex</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>Solutions</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>y-intercept</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
Simplify.

7. $\sqrt{-40}$
8. $\sqrt{5} \cdot \sqrt{-30}$

9. $\sqrt{-150}$
10. $\sqrt{-21} \cdot \sqrt{-35}$

11. $\sqrt{-3} \cdot \sqrt{15}$
12. $8\sqrt{7} \cdot -5\sqrt{-14}$

13. $\sqrt{-6} \cdot \sqrt{-14}$
14. $10\sqrt{-6} \cdot 9\sqrt{-6}$
Solve the system algebraically.
15. \( y = x^2 + 10x + 28 \)
   \( y = -x^2 - 10x - 20 \)
16. \( y = -x^2 + 6x - 15 \)
   \( y = x^2 + 8x + 17 \)

Solve the system by graphing.
17. \( y = x^2 + 13x + 43 \)
   \( y = -(x + 4)^2 + 5 \)
18. \( y = -(x - 6)^2 + 5 \)
   \( y = (x - 1)(x - 5) \)
For each quadratic,

a. Solve using the quadratic formula (must write the formula for each problem)
b. Graph using as many points that fit on the graph
c. Give the x-intercepts
d. Give the y-intercept
e. Name the type and give the coordinate of the critical point

19. \( y = x^2 + 12x + 27 \)

b.

c. \[ \] d. \[ \] e. \[ \]

20. \( y = -2x^2 + 20x - 50 \)

b.

c. \[ \] d. \[ \] e. \[ \]

21. \( y = 3x^2 + 12x + 17 \)

b.

c. \[ \] d. \[ \] e. \[ \]
### Quadratic Review #8

**Supplemental Problems**

**Quadratic Unit**

**Algebra I**

<table>
<thead>
<tr>
<th>Equation in Vertex Form</th>
<th>Vertex</th>
<th>&quot;a&quot; value in vertex form</th>
<th>AOS</th>
<th>Direction of Opening</th>
<th>Critical Point</th>
<th>Graph</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. ( y = (x - 5)^2 + 8 )</td>
<td>(-4, 7)</td>
<td>-1</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>2. ( y = (x - 5)^2 + 8 )</td>
<td>(3, 2)</td>
<td>1</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>3. ( y = (x - 5)^2 + 8 )</td>
<td></td>
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<td></td>
<td></td>
</tr>
<tr>
<td>4.</td>
<td>5.</td>
<td>6.</td>
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<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>Vertex Form</strong></td>
<td>$f(x) = (x + 4)^2 - 4$</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>Standard Form</strong></td>
<td></td>
<td>$y = x^2 + 2x - 3$</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>Factored Form</strong></td>
<td></td>
<td>$f(x) = (x - 1)(x - 5)$</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>LOS</strong></td>
<td></td>
<td></td>
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<td></td>
<td></td>
</tr>
<tr>
<td><strong>Vertex</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>Solutions</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>y-intercept</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>Graph</strong></td>
<td></td>
<td></td>
<td></td>
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<td></td>
<td></td>
</tr>
</tbody>
</table>
Distribute the following.
7. \(4x(3 - 3x + 4)\)
8. \((2x + 5)(3x - 7)\)
9. \((6 - x)^2\)

Factor the following.
10. \(12n^2 + 6mn\)
11. \(x^2 - 5x - 24\)
12. \(3y^2 + 8y + 5\)

Simplify each square root.
13. \(-\sqrt{49}\)
14. \(-\sqrt{150}\)
15. \(3\sqrt{-8} \cdot 4\sqrt{-27}\)
16. \(-5\sqrt{-42} \cdot \sqrt{14}\)

Solve using the Quadratic Formula. Write the answers in coordinate form.
17. \(x^2 + 8x + 7\)
18. \(-x^2 - 10x - 31\)
19. \(-3x^2 - 2x + 5\)
Solve each system by substitution, then graph. You may check your solutions on the calculator. Write the answers as coordinate points.

20. \( y = x^2 + 4x + 4 \)
   \( y = -x^2 - 2x + 4 \)

   Solution(s):________________

21. \( y = x^2 - 2x - 4 \)
   \( y = x^2 + 4x - 4 \)

   Solution(s):________________

22. \( y = x^2 + 4x - 1 \)
   \( y = -x^2 + 6x - 5 \)

   Solution(s):________________
We saw many examples of quadratics in our Families of Functions unit, recall:

- The shape of the graph of a quadratic function is a parabola and is “U” shaped.
- For any given quadratic function, we identified all the critical points as an absolute maximum or absolute minimum. The **vertex** is the absolute maximum or the absolute minimum.
- In graphs of quadratics, the vertex allows us to find the “axis of symmetry.” We could draw an imaginary dotted vertical line cutting the graph in half that goes through this point. The equation of this line is the x-coordinate of the vertex. Write \( x = # \) as the equation of the axis of symmetry.
- We can also determine the x-intercepts (zeros) which are the **solutions** of the quadratic function. They can be found when \( y = 0 \).
- A quadratic can have 0, 1, or 2 solutions.
- We can locate the y-intercept when \( x = 0 \).

1. \( y = (x - 3)^2 - 4 \)
   a. Graph

   ![Graph](image)

   b. Give the coordinate of the vertex. Is the vertex an absolute maximum or minimum?

   c. Identify the solutions (also called zeroes or x-intercepts).

   d. Find the y-intercept.

   e. What is the domain of the function? What is the range of the function?

   f. Draw the axis of symmetry with a dotted line. Give the equation of the axis of symmetry.
1. \( y = -(x + 2)^2 + 9 \)
   a. Graph
   
   b. Give the type and coordinate of the vertex.

   c. Identify the solutions (also called zeroes or x-intercepts).

   d. Find the y-intercept.

   e. What is the domain of the function? What is the range of the function?

   f. Draw the axis of symmetry with a dotted line. Give the equation of the axis of symmetry.

2. \( y = -(x + 1)^2 - 1 \)
   a. Graph

   b. Give the type and coordinate of the vertex.

   c. Identify the solutions (also called zeroes or x-intercepts).

   d. Find the y-intercept.

   e. What is the domain of the function? What is the range of the function?

   f. Draw the axis of symmetry with a dotted line. Give the equation of the axis of symmetry.
Review. Fill in each row from the given information.

<table>
<thead>
<tr>
<th>Graph</th>
<th>Equation</th>
<th>Line of Symmetry</th>
<th>Vertex</th>
<th>Solution(s)</th>
<th>y-intercept</th>
</tr>
</thead>
<tbody>
<tr>
<td><img src="image" alt="Graph" /></td>
<td>$y = (x+2)^2 + 8$</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td><img src="image" alt="Graph" /></td>
<td>$x = 8$</td>
<td></td>
<td>$(6, 0)$</td>
<td>$(10, 0)$</td>
<td>$(0, 60)$</td>
</tr>
<tr>
<td><img src="image" alt="Graph" /></td>
<td></td>
<td></td>
<td>$(4, 0)$</td>
<td></td>
<td>$(0, 16)$</td>
</tr>
</tbody>
</table>
Review. Fill in each row from the given information.

<table>
<thead>
<tr>
<th>Graph</th>
<th>Equation</th>
<th>Line of Symmetry</th>
<th>Vertex</th>
<th>Solution(s)</th>
<th>y-intercept</th>
</tr>
</thead>
<tbody>
<tr>
<td><img src="image" alt="Graph 7" /></td>
<td>$y = (x - 8)^2 - 1$</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td><img src="image" alt="Graph 8" /></td>
<td></td>
<td></td>
<td>(-2, 0)</td>
<td>(0, -4)</td>
<td></td>
</tr>
<tr>
<td><img src="image" alt="Graph 9" /></td>
<td></td>
<td></td>
<td>x = 1</td>
<td>(1, 0)</td>
<td>(0, -1)</td>
</tr>
<tr>
<td><img src="image" alt="Graph 10" /></td>
<td></td>
<td></td>
<td>(7, 3)</td>
<td></td>
<td>(0, 52)</td>
</tr>
</tbody>
</table>
Review. Fill in each row from the given information.

<table>
<thead>
<tr>
<th>Graph</th>
<th>Equation</th>
<th>Line of Symmetry</th>
<th>Vertex</th>
<th>Solution(s)</th>
<th>y-intercept</th>
</tr>
</thead>
<tbody>
<tr>
<td><img src="image" alt="Graph 7" /></td>
<td></td>
<td></td>
<td></td>
<td>(1, 0) (-5, 0)</td>
<td>(0, 5)</td>
</tr>
<tr>
<td><img src="image" alt="Graph 8" /></td>
<td></td>
<td></td>
<td>(-2, -1)</td>
<td></td>
<td>(0, 3)</td>
</tr>
</tbody>
</table>

9. **Test Practice:**

Given the following graph, what is the line of symmetry?

- a. $x = -2$
- b. $x = 0$
- c. $x = -10$
- d. $x = 4$
Complete the table.

<table>
<thead>
<tr>
<th></th>
<th>1.</th>
<th>2.</th>
<th>3.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Vertex Form</td>
<td>f(x) = (x + 3)^2 – 1</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Standard Form</td>
<td></td>
<td>y = x^2 – 8x + 12</td>
<td></td>
</tr>
<tr>
<td>Factored Form</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>LOS</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Vertex</td>
<td></td>
<td>(2, 5)</td>
<td></td>
</tr>
<tr>
<td>Solutions</td>
<td>No x – intercepts</td>
<td></td>
<td></td>
</tr>
<tr>
<td>y-intercept</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>4.</td>
<td>5.</td>
<td>6.</td>
</tr>
<tr>
<td>----------------</td>
<td>-----------------------------</td>
<td>-----------------------------</td>
<td>-----------------------------</td>
</tr>
<tr>
<td>Vertex Form</td>
<td>$f(x) = (x + 1)^2 - 1$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Standard Form</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Factored Form</td>
<td></td>
<td>$f(x) = (x + 5)(x + 1)$</td>
<td></td>
</tr>
<tr>
<td>LOS</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Vertex</td>
<td></td>
<td>$(1, -9)$</td>
<td></td>
</tr>
<tr>
<td>Solutions</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>y-intercept</td>
<td></td>
<td>$(0, -8)$</td>
<td></td>
</tr>
</tbody>
</table>
Now that you have found all forms of each given quadratic and all of the relevant coordinate points, you need to graph each quadratic along with the line of symmetry (LOS). Also, label all relevant points with their coordinates.

**Quadratic #1**

**Quadratic #2**

**Quadratic #3**

**Quadratic #4**

**Quadratic #5**

**Quadratic #6**
Three Methods for Solving Quadratics
Supplemental Problems
Quadratic Unit
Algebra 1

For questions #1-4: Solve by a.) factoring, b.) completing the square, and c.) graphing. When graphing the quadratic, be sure to label the vertex, x-intercept(s) and y-intercept.

1. \( y = x^2 + 12x + 32 \)
   
a.)  
   b.)

2. \( x^2 - 2x + 4 = 3 \)
   
a.)  
   b.)
3. \( x^2 + 6x = -8 \)
   
   a.) 
   
   b.) 

4. \( 0 = x^2 - 8x + 7 \)
   
   a.) 
   
   b.) 

---

RHS Mathematics Department
Algebra 1 Unit 3: Quadratic Functions
For questions #5-8, given the following graphs, write the equation for the function in **factored form, vertex form, and standard form**.

5. [Graph]
   - Factored Form: 
   - Vertex Form: 
   - Standard Form: 

6. [Graph]
   - Factored Form: 
   - Vertex Form: 
   - Standard Form: 

7. [Graph]
   - Factored Form: 
   - Vertex Form: 
   - Standard Form: 

8. [Graph]
   - Factored Form: 
   - Vertex Form: 
   - Standard Form: 

---

RHS Mathematics Department
Algebra 1 Unit 3: Quadratic Functions
Solve each system by substitution. Give coordinates to the nearest hundredth.

1. \[ y = x^2 \]
   \[ y = x^2 + 4x - 12 \]

2. \[ y = -x^2 + 10 \]
   \[ y = x^2 - 8 \]

3. \[ y = -2x^2 + x + 7 \]
   \[ y = x^2 - 2x - 11 \]

4. \[ y = -2x^2 + 2x - 9 \]
   \[ y = -2x^2 \]
5. \( y = 3x^2 + 4x \)  
\( y = 2x^2 - x + 14 \)

6. \( y = 2x^2 - 3x - 2 \)  
\( y = -x^2 + 4x - 6 \)

7. \( y = x^2 - 2x + 2 \)  
\( y = x^2 - 4x + 5 \)

8. \( y = x^2 - 2x + 2 \)  
\( y = \cdot x^2 + 6x - 4 \)
9. \( y = x^2 - x - 4 \)  
   \( y = -x^2 + 4x - 1 \)  

10. \( y = 2x^2 + 16x + 40 \)  
    \( y = -x^2 + 4x - 7 \)  

11. \( y = x^2 + 10x + 19 \)  
    \( y = x^2 + 6x + 7 \)  

12. \( y = 3x^2 - 54x + 239 \)  
    \( y = -x^2 + 18x - 85 \)
1. Simplify the radical expression: \(-4\sqrt{10} \cdot 5\sqrt{15}\)

2. Determine if each function is Linear, Quadratic or Neither. If it is Linear or Quadratic, give the equation.

<table>
<thead>
<tr>
<th>x</th>
<th>y</th>
<th>1st Difference</th>
<th>2nd Difference</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>6</td>
<td></td>
<td></td>
</tr>
<tr>
<td>1</td>
<td>10</td>
<td></td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>18</td>
<td></td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>30</td>
<td></td>
<td></td>
</tr>
<tr>
<td>4</td>
<td>46</td>
<td></td>
<td></td>
</tr>
<tr>
<td>5</td>
<td>66</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

3. Factor: \(64x^2 - 121\)   Factor: \(4x^2 - 4x - 15\)
4. Given the function \( y = -(x + 3)^2 + 4 \), graph and list the zeros and critical point.

\[ \text{Zeros: } \quad \text{CP: } \]

5. Solve the following quadratic equation by factoring: \( f(x) = x^2 - x - 6 \)

6. Complete the square to write \( y = x^2 + 6x - 12 \) in vertex form. Also, state the vertex of the function.
7. Solve for $x$: \[ 5(x - 12)^2 - 4 = 41 \]

8. Solve the given equation for $x$: \[ -3x^2 - 81 = 0 \]

9. Given the table of values, identify the correct quadratic equation in standard form (the parabola opens down).

<table>
<thead>
<tr>
<th>$x$</th>
<th>-4</th>
<th>-3</th>
<th>-2</th>
<th>-1</th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
</tr>
</thead>
<tbody>
<tr>
<td>$y$</td>
<td>-6</td>
<td>0</td>
<td>4</td>
<td>6</td>
<td>6</td>
<td>4</td>
<td>0</td>
<td>-6</td>
</tr>
</tbody>
</table>

10. Simplify completely: \[ 5 \cdot \sqrt{-68} \]
11. Solve $x^2 - 3x - 10$ using the quadratic formula. Verify the solution by graphing.

12. Given the following two tables, fill in the blanks.

a.)

<table>
<thead>
<tr>
<th>Vertex Form</th>
<th>$y = (x - 2)^2 - 9$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Standard Form</td>
<td></td>
</tr>
<tr>
<td>Factored Form</td>
<td></td>
</tr>
<tr>
<td>LOS</td>
<td></td>
</tr>
<tr>
<td>Vertex</td>
<td></td>
</tr>
<tr>
<td>Solution(s)</td>
<td></td>
</tr>
<tr>
<td>y-intercept</td>
<td></td>
</tr>
<tr>
<td>Direction of Opening</td>
<td></td>
</tr>
</tbody>
</table>

b.)
13. Solve the given quadratic system by substitution.
   \[ y = 2x^2 - 4 \]
   \[ y = -x^2 + x - 2 \]

14. Graph the following quadratic equations.
   a. \( f(x) = x^2 - 4x + 3 \)
   b. \( y = (x - 5)^2 + 3 \)
   c. \( f(x) = (x + 3)(x - 1) \)
   d. \( y = x^2 + 3x - 4 \)
15. The quadratic opens down, and has \((3,0)\) as the only solution. Write the equation for the quadratic function in factored form and standard form.

For questions #16 – 20 use the following function: \(y = -(x + 2)^2 + 4\)

16. Write the function in standard form.

17. Write the function in factored form.

18. What are the zeros of the function?
19. What is the y-intercept?

20. What is the equation for the LOS (line of symmetry) for the function?

21. Solve the following quadratic system by graphing:

\[
\begin{align*}
y &= x^2 - 6x + 8 \\
y &= -(x - 4)^2 + 4
\end{align*}
\]

22. Solve the quadratic function \( x^2 - 6x + 5 = 0 \) by completing the square.

23. Solve the following inequality: \( y < -(x + 5)^2 + 4 \)
Pine Knob currently charges $30 for a weekend lift ticket from 9:00 AM until 5:30 PM. They want to expand their facility by adding to the lodging portion of the ski resort. To do this they need to raise more money. They have found an increased interest among the high school students in the winter sport of skiing and snow boarding. They decide to poll high school students to see how much they would be willing to pay for a lift ticket. They find:

- At the current rate of $30, they get 215 visitors on average per Saturday.
- For every $4.00 they raise the price, they will lose 10 visitors.

24. Fill in the table with the appropriate information.

<table>
<thead>
<tr>
<th>x</th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
</tr>
</thead>
<tbody>
<tr>
<td>Attendance (A)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Ticket Price (T)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Revenue (R)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

25. Find the equation that would be used to calculate the lift ticket price, attendance, and revenue for Pine Knob on a Saturday.
26. Determine the # of price changes, revenue, attendance, and ticket price when the revenue is the highest.